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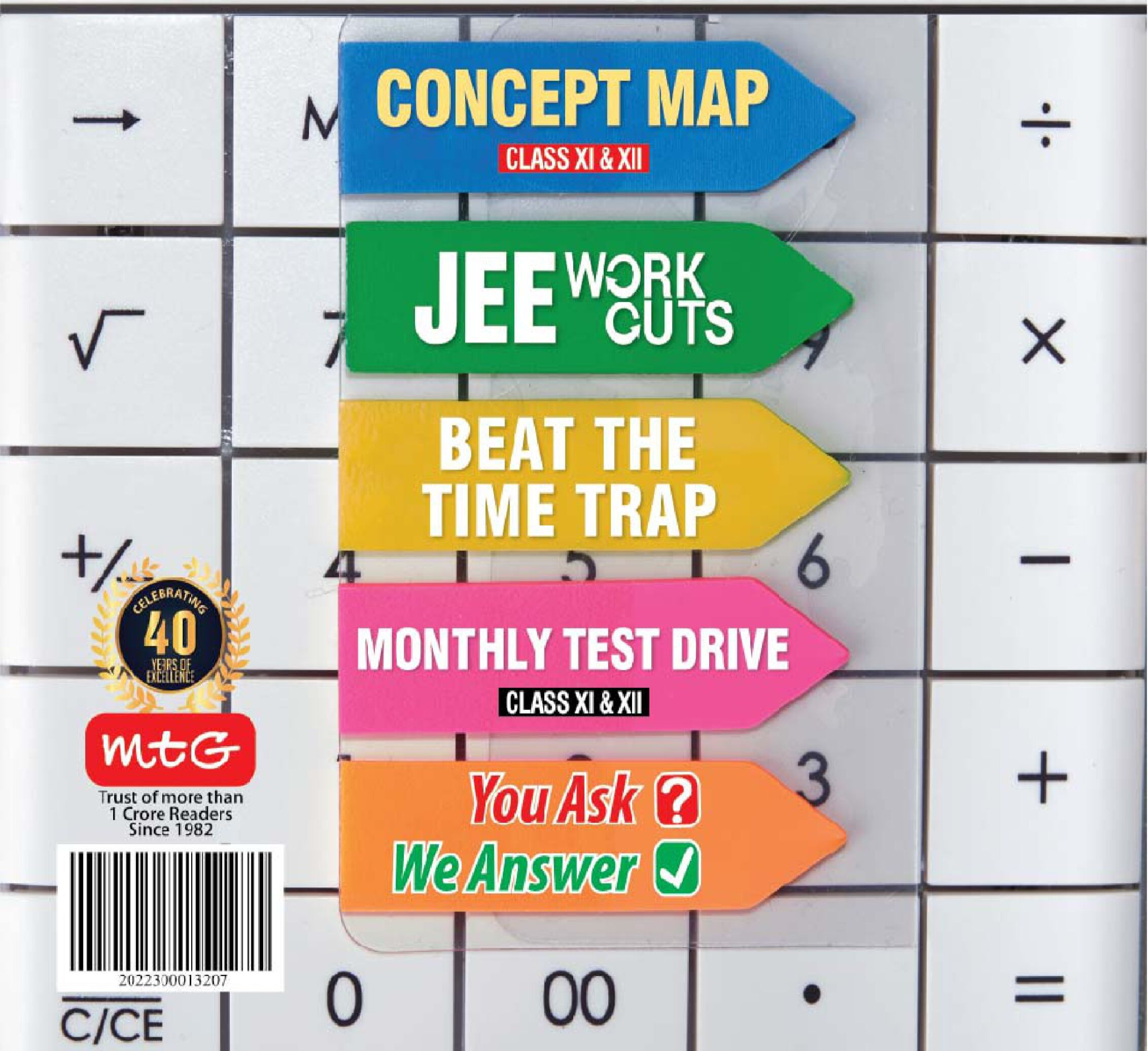

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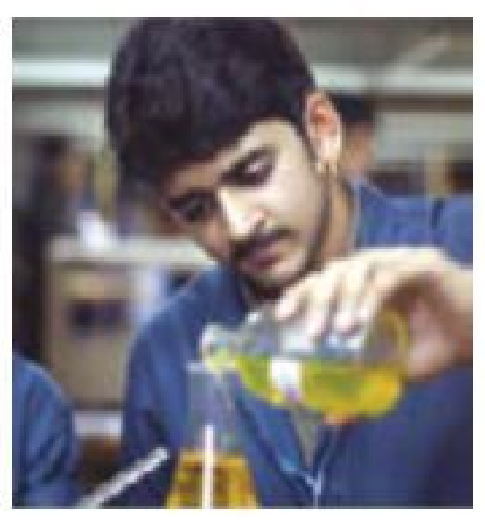

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Tips on Revision

Preparing for JEE and NEET is a lot of work. But you know that already. Did you know revision can be a little less work if you plan right? Find out how.

Our memory requires a bit of a refresher every once in a while. Be it a favourite novel, a place or academics. Especially academics, and more so if you are appearing for JEE or NEET. So, all those students dreaming of donning the scrubs should know that revision is an important and integral part of preparing for JEE and NEET.

You must know that along with efficient preparation, strategic revision is equally important. Here are some tips for JEE and NEET revision:

1. Make a Proper Schedule

Plan ahead, chalk out a proper schedule for revision. Allot sufficient time to each topic and subject based on the time you have left before the exam. A good idea is to start with topics that you find tough so that you can devote more time to them. An ideal way to ensure that you do this is by preparing a list of topics segregated into three categories: Need a lot of work, Need some work, Need to flip through. You can then designate the number of days/hours that each of these topics need and take it from there.

2. Go through the syllabus, exam pattern

Keep tab on the exam pattern and syllabus so that you can prepare accordingly. This is crucial because the syllabus or pattern might have changed since you started preparation! So be alert and double check.

3. Clear basic concepts of numerical-related topics

While NEET is not as concept-oriented as some other undergraduate exams, and JEE is more concept oriented. Physics and Chemistry still rely heavily on conceptual grasp of all topics. Numerical-based questions in these subjects, especially Physics, can be solved easily if your concepts are clear. You can score well in JEE and NEET with an above average understanding of basic concepts.

4. Compile all revision resources

You would need NCERT books of Physics, Chemistry, Math and Biology ready, as well as previous year question papers and notes that you have prepared earlier. Form a habit of marking important or difficult questions. This will come handy while revising.

5. Take Mock Tests

Taking mock tests is the best way to understand your last-minute preparation level. Solving questions of different difficulty levels along with previous year papers, mock tests, and practice sheets should form an integral part of your daily revision routine. Taking tests regularly also reduces the time it usually takes you to answer a question. This in turn will improve your score in the time-bound JEE and NEET examination.

6. Do not prioritise favourites

Every student has his or her favourite subject/s. It's easy to study those over other tougher ones. However, for a competitive exam like JEE or NEET, all the subjects need to studied with equal dedication. A way out of this is to devise a timetable in such a way that you alternate between your favourite subjects with the ones you find difficult while revising.

7. Do not take up new topics

You need to know how to utilise your time just before the exam. There are chances that you may have missed studying certain topics. Now is not the time to pick those up. Taking up a new topic will only be a waste of time. Be smart. Focus on strengthening the topics that you are well-versed with.

8. Prepare a formula sheet

For Physics and Chemistry, a pre-prepared formula sheet goes a long way in easy revision. Use it. It helps avoid glitches during the examination. Add new formulas to your sheet as and when you study a topic, instead of spending a huge amount of time in the end on just preparing the formula sheet. This will help you memorise and retain your study material better.

9. Limit distractions

Distractions happen when fatigue sets in. Try to limit those including scrolling aimlessly on social media websites. They tend to be addictive. Instead, catch an episode of your favourite series, or go for a walk when you feel mentally exhausted.

Studying non-stop is known to hinder performance. Take short breaks. These help you relax your mind, improve concentration and retention. A calm mind is better than a tired one. We hope you can incorporate some or all of the tips to help you revise better, smarter.

All the best!

Disclaimer: The information provided in this article is of a general nature and should not be considered a substitute for professional advice.

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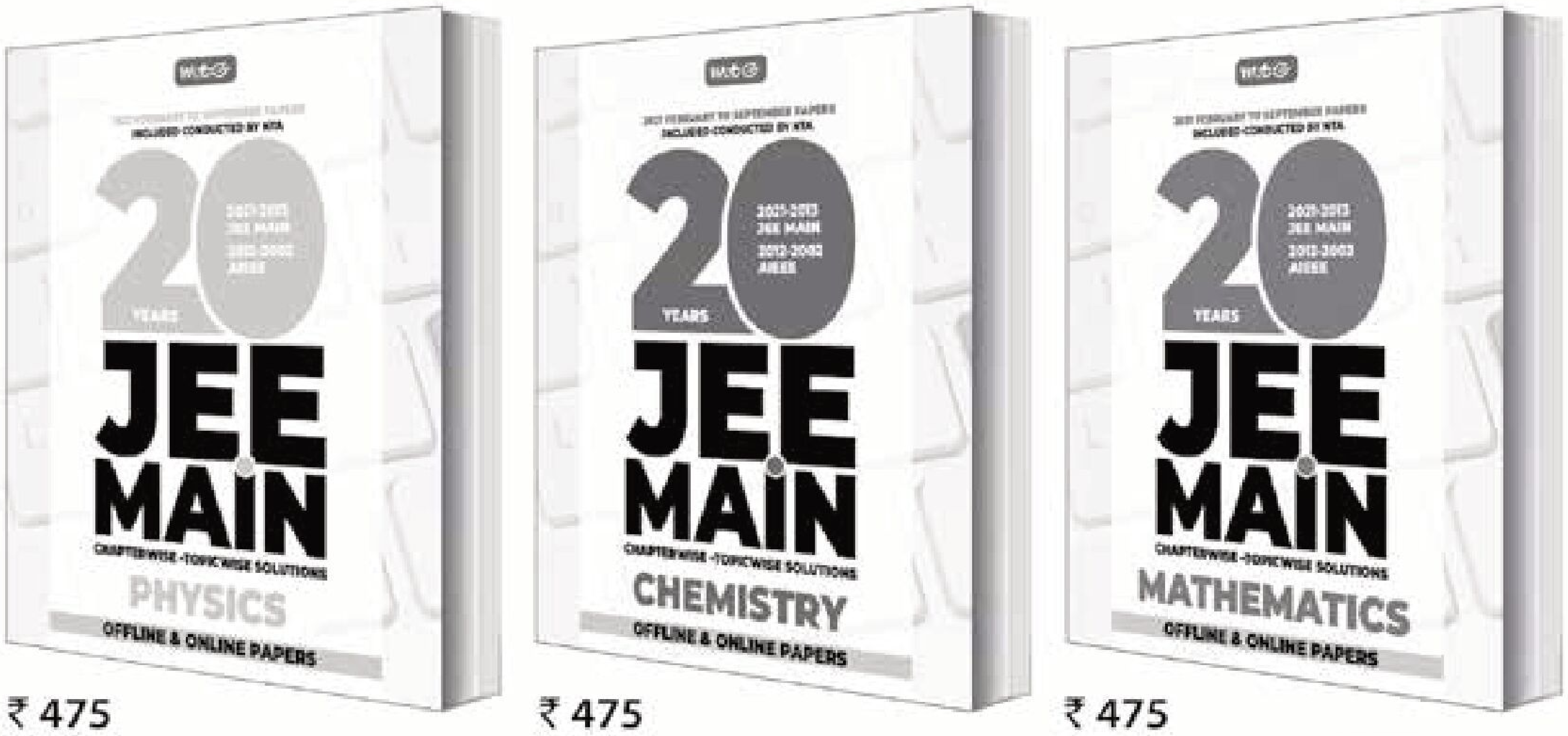
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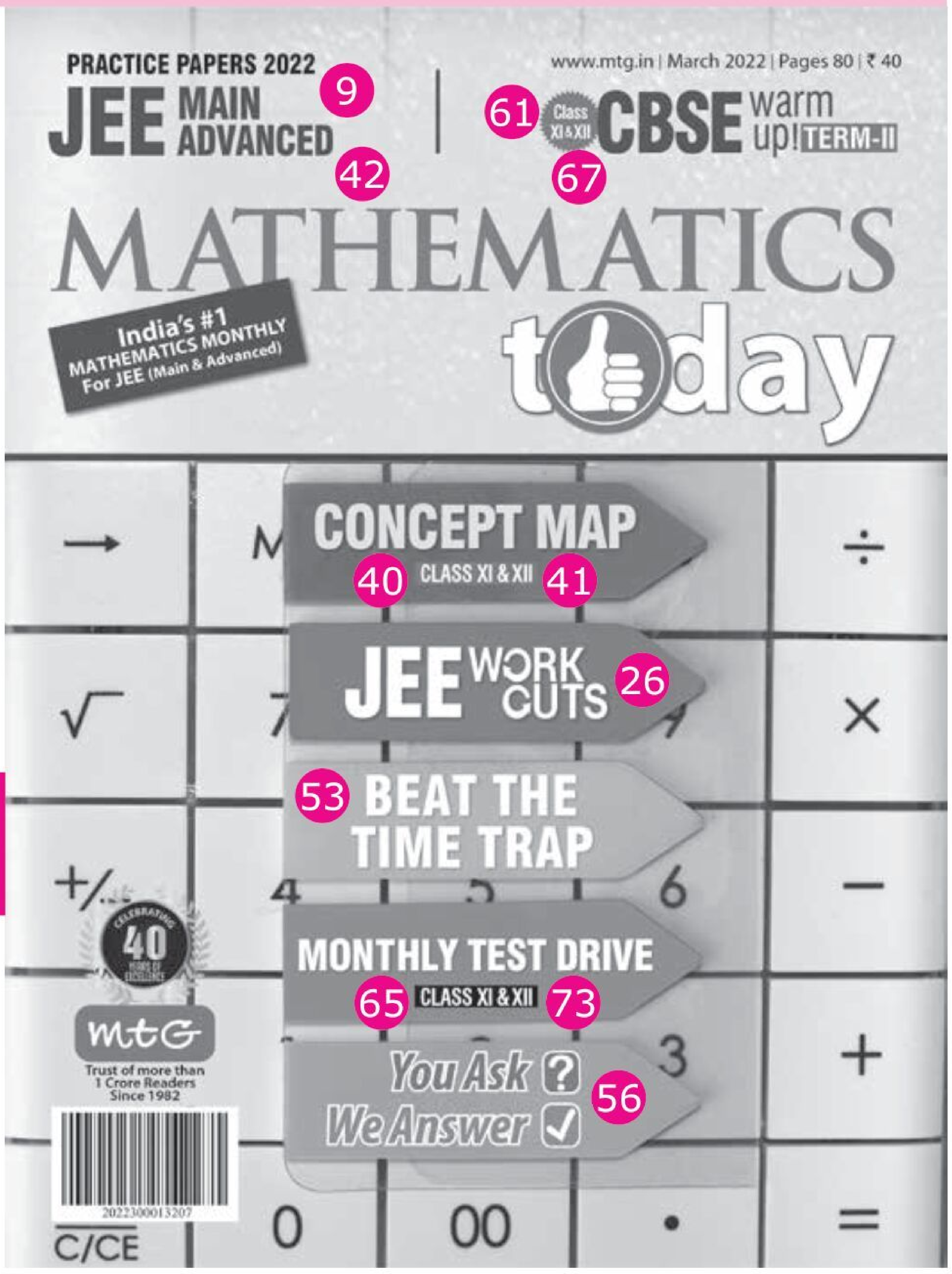
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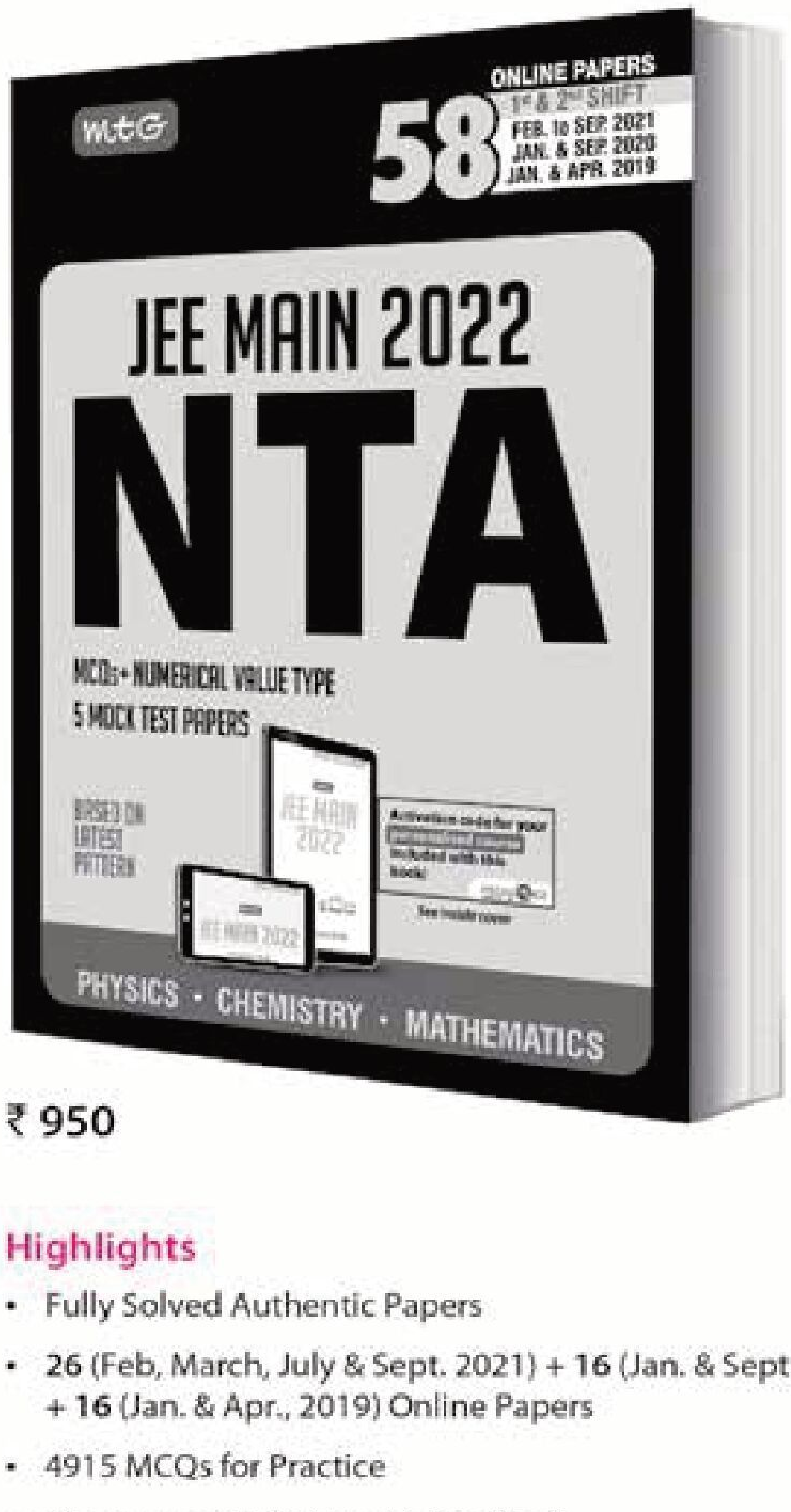
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SINGLE OPTION CORRECT TYPE

1. If $\lim\limits_{x\to 0}\frac{4}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cdot\cos\frac{x^2}{4}\right)=\left(\frac{1}{n_1}\right)^n$ where n_1 is a prime number, then value of $n_1 + n$ equals
(a) 8 (b) 6 (c) 4 (d) 2

2. Let the unit vectors $\vec{a}$ and $\vec{b}$ be perpendicular to each other and the unit vector $\vec{c}$ be inclined at an angle 'θ' to both $\vec{a}$ and $\vec{b}$. If $\vec{c}=\alpha\,\vec{a}+\beta\,\vec{b}+\gamma\,(\vec{a}\times\vec{b})$ then,
(a) $\alpha = \beta = \cos\theta, \gamma^2 = \cos 2\theta$
(b) $\alpha = \beta = \cos\theta, \gamma^2 = -\cos 2\theta$
(c) $\alpha = \cos\theta, \beta = \sin\theta, \gamma^2 = \cos 2\theta$
(d) $\alpha = \sin\theta, \beta = \cos\theta, \gamma^2 = -\cos 2\theta$

3. Each of m urns consisting 6 red and 8 black balls. The $(m + 1)^{\text{th}}$ urn consisting 7 red and 7 black balls. One of the $(m + 1)$ urns is selected randomly and two balls are drawn from it without replacement and found to be black. If the probability that $(m + 1)^{\text{th}}$ urn was selected to draw the ball is $\frac{1}{17}$, then the value of m equal
(a) 16 (b) 15 (c) 12 (d) 10

4. The minimum & maximum distance of a point $(-1, 6)$ from the ellipse $25x^2 + 9y^2 + 50x + 36y - 164 = 0$ are l and m respectively then $m^2 - l^2$ equals
(a) 169 (b) 13 (c) 100 (d) 160

5. If $2f(x)+xf\left(\frac{1}{x}\right)-2f\left[\left|\sqrt{2}\sin\pi\left(x+\frac{1}{4}\right)\right|\right]$

$$=4\cos^2\left(\frac{\pi x}{2}\right)+x\cos\frac{\pi}{x}$$

$\forall\, x \in R - \{0\}$, then which of the following is false?
(a) $f(2)+f\left(\frac{1}{2}\right)=1$ (b) $f(2)+f(1)=f\left(\frac{1}{2}\right)$
(c) $f(2) + f(1) = 0$ (d) $f(1)f\left(\frac{1}{2}\right)f(2)=1$

6. In a bag there are 10 balls of either Black or White. Let A_k be the event which contains exactly k white balls and its probability is proportional to k^2. Now an another ball is drawn randomly. If $P(E)$ be the probability that the ball drawn is white and $P(E)$ $=\frac{a}{b}$ where HCF $(a, b) = 1$, then the value of $|a - b|$ equals
(a) 14 (b) 11 (c) 8 (d) 3

7. Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$. If plane P divides the line segment AB joining points $A(-3, -6, 1)$ and $B(2, 4, -3)$ in ratio $k : 1$, then the value of k is equal to
(a) 1.5 (b) 4 (c) 3 (d) 2

8. If $(x^2 + x + 3)^2 - (p - 3)(x^2 + x + 3)(x^2 + x + 2) + (p - 4)(x^2 + x + 2)^2 = 0$ has at least one real root, then range of p is
(a) $\left[-5, \frac{39}{7}\right]$ (b) $\left(5, \frac{39}{7}\right]$
(c) $\left(5, -\frac{39}{7}\right)$ (d) None of these

9. A function f defined for all positive integers and satisfies $f(1) = 4031$ and $f(1) + f(2) + \ldots + f(n)\}$ $= n^2f(n)\ \forall\ n > 1$, then find the value of $\frac{1}{f(4030)}$.
(a) 2013 (b) 2014 (c) 2015 (d) 2016

10. If n is the degree of the polynomial,

$$\left[\frac{2}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8+\left[\frac{2}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$$

and m is the coefficient of x^n in it, then the ordered pair (n, m) is equal to
(a) $(24, (10)^8)$ (b) $(12, (20)^4)$
(c) $(12, 8(10)^4)$ (d) $(8, 5(10)^4)$

11. The value of $\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$ is

(a) $\frac{23}{22}$ (b) $\frac{22}{23}$ (c) $\frac{19}{21}$ (d) $\frac{21}{19}$

12. Let $a, b, c \in R$ such that $abc = 2$ if $A = \begin{pmatrix} 3a & b & c \\ b & 3c & a \\ c & a & 3b \end{pmatrix}$ is such that $AA^T = 16I$ and $|A|$ is a positive real number, then the value of $(a^3 + b^3 + c^3)^2$ equals

(a) 9 (b) 16
(c) 4 (d) None of these

13. If the edges of a rectangular parallelepiped be a, b, c, then the angles between the four diagonals are given by

(a) $\cos^{-1}\left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2+b^2+c^2}\right)$

(b) $\cos^{-1}\left(\frac{a^2+b^2+c^2}{a^2+b^2+c^2}\right)$

(c) $\cos^{-1}\left(\frac{-a^2+b^2+c^2}{a^2+b^2+c^2}\right)$

(d) None of these

14. If both the mean and the standard deviation of 50 observations $x_1, x_2, ..., x_{50}$ are equal to 16, then the mean of $(x_1 - 4)^2, (x_2 - 4)^2, ..., (x_{50} - 4)^2$ is :

(a) 525 (b) 380 (c) 480 (d) 400

15. If $z_n = \text{cis}\left(\frac{\pi}{n(n+1)(n+2)}\right)$ for n = 1, 2, 3, ... and $z = \lim_{n\to\infty}(z_1 z_2 \ldots z_n)$, then principal argument of $z = \lim_{n\to\infty}(z_1, \ldots, z_n)$ is

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{2\pi}{3}$

16. Let P be a point on the parabola, $y^2 = 12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then

(a) $PN = 4$ (b) $MQ = \frac{1}{3}$

(c) $MQ = \frac{1}{4}$ (d) $PN = 3$

17. If $\int \sin^{-1}\left(\sqrt{\frac{x}{1+x}}\right)dx = A(x)\tan^{-1}(\sqrt{x}) + B(x) +$ C, where C is a constant of integration, then the ordered pair $(A(x), B(x))$ can be

(a) $(x+1, -\sqrt{x})$ (b) $(x+1, \sqrt{x})$
(c) $(x-1, -\sqrt{x})$ (d) $(x-1, \sqrt{x})$

18. The cubes of the natural numbers are grouped as $1^3, (2^3, 3^3), (4^3, 5^3, 6^3)$ ... then sum of the numbers in the n^{th} group is

(a) $\frac{n^2}{12}(n^2 + 1)(n^2 + 4)$ (b) $\frac{n^3}{8}(n^2 + 1)(n^2 + 3)$

(c) $\frac{n^3}{8}(n^2 + 2)(n^2 + 4)$ (d) $\frac{n^3}{16}(n^2 + 1)(n^2 + 4)$

19. If the solution of differential equation $\frac{dy}{dx} = \frac{mx+5}{4y+f}$ represents a circle, then the values of m & radius r of circle respectively satisfies

(a) $m = -4, r = \frac{1}{4}\sqrt{f^2 + 8k + 25}$

(b) $m = 4, r = \frac{1}{4}\sqrt{f^2 + 8k + 100}$

(c) $r = 3$, $m = 3$

(d) $m = -4, r = \frac{1}{16}\sqrt{f^2 + 8k - 100}$

20. Let the variate takes the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_n$ respectively. If mean is $\frac{91\times 8}{2^n}$, then n equals

(a) 4 (b) 6
(c) 5 (d) None of these

NUMERICAL ANSWER TYPE

21. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 k, then k is equal to _______.

22. If s and p be sum and product of slopes of 2 lines represented by $11x^2 - 100xy + 5y^2 = 0$, then $5(p + 20s) =$ _______.

23. The area of the region $S = \{(x, y) : 3x^2 \le 4y \le 6x + 24\}$ is _______.

24. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles' equations

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 24x - 10y + 160 = 0$ is _______.

25. Let z_0 be a root of the quadratic equation, $x^2+x+1=0$. If $z=3+6iz_0^{81}-3iz_0^{93}$, and arg($z$) is equal to $m°$, then the value of $m°$ is _____.

26. Let $f: R \to R$ be defined as

$$f(x)=\begin{cases} x^5\sin\left(\frac{1}{x}\right)+5x^2 & , x<0 \\ 0 & , x=0 \\ x^5\cos\left(\frac{1}{x}\right)+\lambda x^2 & , x>0 \end{cases}$$

The value of λ for which $f''(0)$ exists, is _________.

27. The number of integral values of k for which the line, $3x + 4y = k$ intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is _____.

28. Let a, b, c are distinct integers such that $c = b + 1$ and $b = a + 1$ and ω is the cube root of unity other than 1. If minimum value of $|a + b\omega + c\omega^2| + |a + b\omega^2 + c\omega|$ is $\alpha^{1/6}$, then α equals to ______ .

29. The number of 4 letter words (with or without meaning) that can be formed from the eleven letters of the word 'Examination' is ______.

30. A tangent is drawn to the curve $y = f(x)$ at $P(x, y)$ cuts the x and y axes at A and B respectively such that $AP : PB = 1 : 3$. If $f(1) = 1$, then the curve passes through $\left(k, \frac{1}{8}\right)$, where $k =$ _____ .

SOLUTIONS

1. (a): $\lim\limits_{x\to 0}\frac{4}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cdot\cos\frac{x^2}{4}\right)$

$$=\lim_{x\to 0}\frac{4}{x^8}\left\{\left(1-\cos\frac{x^2}{2}\right)-\cos\frac{x^2}{4}\left(1-\cos\frac{x^2}{2}\right)\right\}$$

$$=\lim_{x\to 0}4\left\{\frac{\left(1-\cos\frac{x^2}{2}\right)}{x^4}\right\}\left\{\frac{\left(1-\cos\frac{x^2}{4}\right)}{x^4}\right\}$$

$$=\lim_{x\to 0}16\left(\frac{\sin\frac{x^2}{4}}{4\frac{x^2}{4}}\right)^2\times\left(\frac{\sin\frac{x^2}{8}}{8\frac{x^2}{8}}\right)^2$$

$$=\frac{16}{4^2\times 8^2}\lim_{x\to 0}\left(\frac{\sin\frac{x^2}{4}}{\frac{x^2}{4}}\right)^2\cdot\lim_{x\to 0}\left(\frac{\sin\frac{x^2}{8}}{\frac{x^2}{8}}\right)^2$$

$4\cos^2\left(\frac{\pi x}{2}\right)+$

Now, $\frac{1}{2^6}=\left(\frac{1}{n_1}\right)^n \Rightarrow \left(\frac{1}{2}\right)^6=\left(\frac{1}{n_1}\right)^n$

$\therefore \quad n_1 + n = 2 + 6 = 8$

2. (b): $\vec{a}\cdot\vec{b}=0,\ \vec{a}\cdot\vec{c}=\cos\theta,\ \vec{b}\cdot\vec{c}=\cos\theta$

($\because\ \vec{a},\vec{b},\vec{c}$ are unit vectors)

Also, $\vec{c}=\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})$

$\therefore \quad \vec{c}\cdot\vec{a} = \alpha+\beta\cdot 0+\gamma(\vec{a}\times\vec{b})\cdot\vec{a} \Rightarrow \cos\theta = \alpha$

Similarly $\beta = \cos\theta$

Now, $\gamma(\vec{a}\times\vec{b})=\vec{c}-\alpha\vec{a}-\beta\vec{b}$

$\Rightarrow|\gamma(\vec{a}\times\vec{b})|^2=|\vec{c}|^2+\alpha^2|\vec{a}|^2+\beta^2|\vec{b}|^2-2\alpha\vec{a}\cdot\vec{c}-2\beta\vec{b}\cdot\vec{c}$

$\Rightarrow \quad \gamma^2 = 1 + \alpha^2 + \beta^2 - 2\alpha\cos\theta - 2\beta\cos\theta$

$\Rightarrow \quad \gamma^2 = 1 - 2\cos^2\theta = -\cos 2\theta$

3. (c): Given, each m urns having 6 red and 8 black balls and $(m + 1)^{th}$ urn consists 7 red and 7 black balls. Let A_1 and A_2 denotes the events that one of first m urns and $(m + 1)^{th}$ urn is selected respectively.

$\therefore \quad P(A_1)=\frac{m}{m+1},\ P(A_2)=\frac{1}{m+1}$

Let A be the event that two balls drawn without replacement from the urn are black.

$\therefore \quad P(A|A_1)=\frac{^8C_2}{^{14}C_2}=\frac{8}{14}\times\frac{7}{13}=\frac{4}{13}$

and $P(A|A_2)=\frac{^7C_2}{^{14}C_2}=\frac{7}{14}\times\frac{6}{13}=\frac{3}{13}$

$\therefore \quad P(A)=P(A_1)P(A|A_1)+P(A_2)P(A|A_2)$

$\therefore \quad P(A)=\left(\frac{m}{m+1}\right)\left(\frac{4}{13}\right)+\frac{1}{(m+1)}\left(\frac{3}{13}\right)=\frac{4m+3}{13(m+1)}$

$\therefore$ By the Baye's theorem, we have

$$P(A_2\,|\,A)=\frac{P(A_2)P(A\,|\,A_2)}{P(A)}=\frac{\left(\frac{1}{m+1}\right)\left(\frac{3}{13}\right)}{\frac{4m+3}{13(m+1)}}$$

$\therefore\quad \frac{1}{17}=\frac{3}{4m+3}\;\Rightarrow\; 4m+3=51\;\Rightarrow\; m=12.$

4. (d): Let $S=25x^2+9y^2+50x+36y-164=0$ ...(i)

$\therefore\quad S_{(-1,\,6)}=25(-1)^2+9(6)^2+50(-1)+36(6)-164$

$=25+324-50+216-164>0$

which means the point $(-1, 6)$ is outside the ellipse.

Now, (i) can be rewritten as

$$\frac{(x+1)^2}{9}+\frac{(y+2)^2}{25}=1=\frac{X^2}{a^2}+\frac{Y^2}{b^2}$$

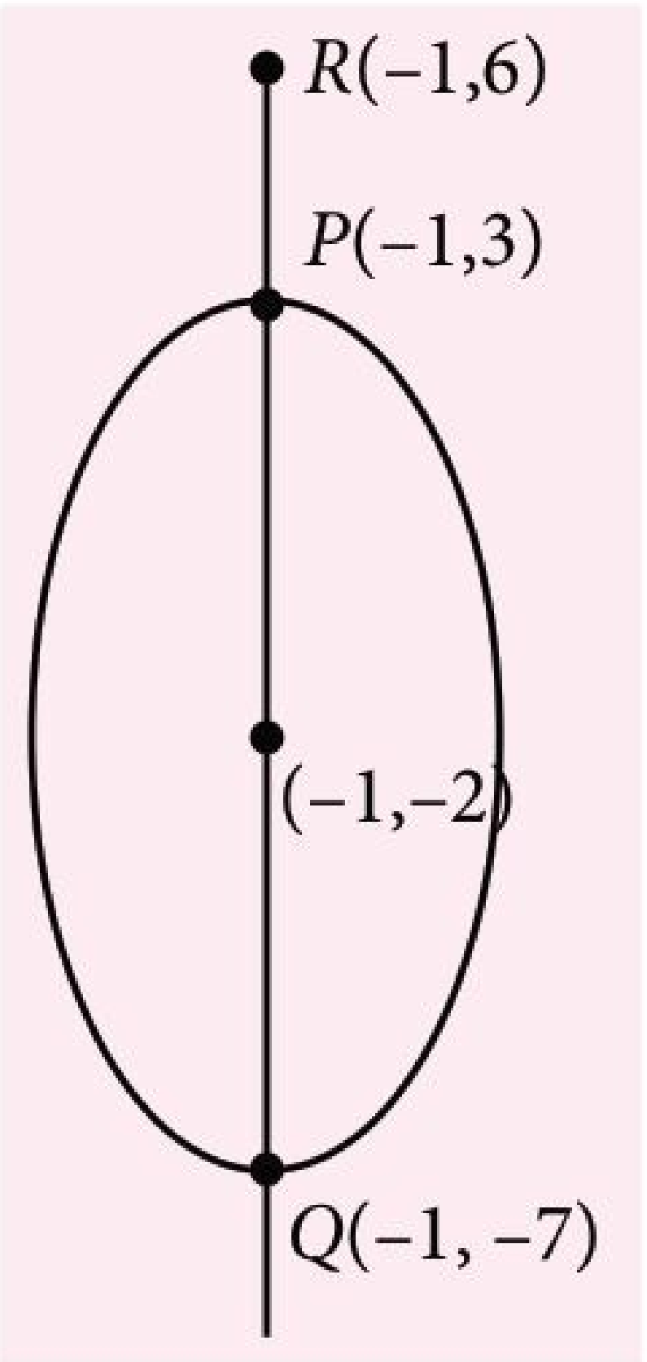
$\therefore$ Centre of ellipse is $(-1,-2)$

& axis is parallel to y-axis as $b>a$.

$\therefore$ Vertices are $x+1=0$ & $y+2=\pm 5$ *i.e.*, $(-1,3)$ and $(-1,-7)$

The minimum distance $=l=RP=6-3=3$ units and the maximum distance $=m=RQ=6-(-7)=13$ units

$\therefore\quad m^2-l^2=13^2-3^2=160$



5. (d): $$2f(x)+xf\left(\frac{1}{x}\right)-2f\left[\left|\sqrt{2}\sin\pi\left(x+\frac{1}{4}\right)\right|\right]=4\cos^2\left(\frac{\pi x}{2}\right)+x\cos\left(\frac{\pi}{x}\right)\quad\ldots\text{(i)}$$

When $x=2$, we have

$$2f(2)+2f\left(\frac{1}{2}\right)-2f\left[\left|\sqrt{2}\sin\frac{\pi}{4}\right|\right]=4\cos^2(\pi)+2\cos\frac{\pi}{2}$$

$$\Rightarrow\; 2f(2)+2f\left(\frac{1}{2}\right)-2f(1)=4+0$$

$$\Rightarrow\; f(2)+f\left(\frac{1}{2}\right)=f(1)+2\quad\ldots\text{(ii)}$$

When $x=1$, we have

$2f(1)+f(1)-2f(|-1|)=-1\;\Rightarrow\; f(1)=-1$ ...(*)

From (ii), we get $f(2)+f\left(\frac{1}{2}\right)=1$ ...(iii)

When $x=\frac{1}{2}$ then from (i), we get

$$2f\left(\frac{1}{2}\right)+\frac{1}{2}f(2)-2f(|1|)=4\left(\frac{1}{2}\right)+\frac{1}{2}$$

$$\Rightarrow\; 2f\left(\frac{1}{2}\right)+\frac{1}{2}f(2)+2=2+\frac{1}{2}$$

$$\Rightarrow\; 2f\left(\frac{1}{2}\right)+\frac{1}{2}f(2)=\frac{1}{2}\quad\ldots\text{(iv)}$$

Solving (iii) & (iv), we get $f\left(\frac{1}{2}\right)=0$ and $f(2)=1$

as $f(2)=1\Rightarrow f(2)=-f(1)$ (using (*))

$\Rightarrow\; f(2)+f(1)=0$

6. (d): Given $P(A_k)\propto k^2\;\therefore\; P(A_k)=\lambda k^2$

As the event are mutually exclusive and exhaustive

$$\therefore\quad \sum_{k=0}^{n}P(A_k)=1\;\Rightarrow\;\lambda\sum_{k=0}^{n}k^2=1$$

$$\because\quad \lambda(1^2+2^2+\ldots+n^2)=1\Rightarrow\lambda=\frac{6}{n(n+1)(2n+1)}\quad\ldots(*)$$

Now $P(E)$ be the probability that the ball drawn is white

$$\therefore\quad P(E)=\sum_{k=0}^{n}P(A_k)P\left(\frac{E}{A_k}\right)$$

$$\Rightarrow\; \frac{a}{b}=\sum_{k=0}^{n}\lambda k^2\left(\frac{k}{n}\right)$$

$$\because\quad \frac{a}{b}=\frac{\lambda}{n}\left[\sum_{k=0}^{n}k^3\right]=\frac{\lambda}{n}\cdot\frac{n^2(n+1)^2}{4}$$

$$=\frac{6n^2(n+1)^2}{4n^2(n+1)(2n+1)}=\frac{3(n+1)}{2(2n+1)}\qquad\text{(using (*))}$$

Putting $n=10$, we have

$$\frac{a}{b}=\frac{3(11)}{2\times 21}=\frac{33}{42}=\frac{11}{14}\quad\therefore\;|a-b|=3$$

7. (d): The given plane is $lx+my+nz=0$ ...(i)

and line is

$$\frac{x-1}{-1}=\frac{y-(-4)}{2}=\frac{z-(-2)}{3}\quad\ldots\text{(ii)}$$

Now, plane (i) containing the line (ii), therefore

$-l+2m+3n=0$ ...(iii) and $l-4m-2n=0$ ...(iv)

Solving (iii) and (iv), we get $\frac{l}{-4+12}=\frac{m}{3-2}=\frac{n}{4-2}$

$\Rightarrow\quad l:m:n=8:1:2$

So, equation of plane is $8x+y+2z=0$

Now, let the plane divides the line joining A and B at point C in the ratio $k:1$. Then,

$$\text{Coordinates of } C\equiv\left(\frac{2k-3}{k+1},\frac{4k-6}{k+1},\frac{-3k+1}{k+1}\right)$$

which satisfies the equation of plane.

$$\Rightarrow\; 8\left(\frac{2k-3}{k+1}\right)+\left(\frac{4k-6}{k+1}\right)+2\left(\frac{-3k+1}{k+1}\right)=0$$

$\Rightarrow\quad 14k-28=0\;\Rightarrow\; k=2$

8. (b): The given equation can be written as

$$\left(\frac{x^2+x+3}{x^2+x+2}\right)^2-(p-3)\left(\frac{x^2+x+3}{x^2+x+2}\right)+(p-4)=0.$$

$$\Rightarrow \quad R^2-(p-3)R+(p-4)=0 \qquad \ldots(*)$$

where $R=\dfrac{x^2+x+3}{x^2+x+2}=1+\dfrac{1}{x^2+x+2}\ge 1$

Now, R will be maximum if (x^2+x+2) is minimum and minimum of x^2+x+2 is $\dfrac{7}{4}$.

$$\therefore \quad \text{Maximum of } R=1+\frac{1}{7/4}=\frac{11}{7}$$

$$\therefore \quad R\in\left[1,\frac{11}{7}\right] \text{ i.e. } 1\le R\le\frac{11}{7}$$

Now, at least one root of (*) must lies in $1\le R\le\dfrac{11}{7}$.

Now, roots of the equation (*) are $p-4$ and 1.

We must have, $1<p-4\le\dfrac{11}{7}$

$$\Rightarrow \quad 5<p\le\frac{11}{7}+4 \Rightarrow 5<p\le\frac{39}{7}.$$

$\Rightarrow$ Minimum value of p is greater than 5 and Maximum value of p is $\dfrac{39}{7}$.

$$\therefore \quad p\in\left(5,\frac{39}{7}\right]$$

9. (c) : Given, $f(1)=4031$

and $n^2f(n)=f(1)+f(2)+f(3)+\ldots+f(n),\ \forall\ n>1$

When $n=2$, then $4f(2)=f(1)+f(2)$

$$\Rightarrow f(2)=\frac{f(1)}{3}=\frac{f(1)}{1+2}$$

When $n=3$, then $9f(3)=f(1)+f(2)+f(3)$

$$\Rightarrow \quad 8f(3)=f(1)+f(2)=f(1)+\frac{f(1)}{3}$$

$$\therefore \quad f(3)=\frac{f(1)}{6}=\frac{f(1)}{1+2+3}$$

....

....

$$\therefore \quad f(4030)=\frac{f(1)}{1+2+3+\ldots\ldots+4030}=\frac{2\times f(1)}{4030\times 4031}$$

$$\therefore \quad f(4030)=\frac{2\times 4031}{4030\times 4031}=\frac{1}{2015}$$

$$\therefore \quad \frac{1}{f(4030)}=2015$$

10. (b): On rationalizing given polynomial, we get

$$\left[\frac{2(\sqrt{5x^3+1}+\sqrt{5x^3-1})}{2}\right]^8+\left[\frac{2(\sqrt{5x^3+1}-\sqrt{5x^3-1})}{2}\right]^8$$

$$=2[{}^8C_0(\sqrt{5x^3+1})^8+{}^8C_2(\sqrt{5x^3+1})^6(5x^3-1)+{}^8C_4(\sqrt{5x^3+1})^4(5x^3-1)^2+{}^8C_6(\sqrt{5x^3+1})^2(5x^3-1)^3+{}^8C_8(5x^3-1)^4]$$

$$=2[(5x^3+1)^4+28(5x^3+1)^3(5x^3-1)+70(5x^3+1)^2(5x^3-1)^2+28(5x^3+1)(5x^3-1)^3+(5x^3-1)^4]$$

$\therefore \quad n=12$

and $m=2(5^4+140\cdot 5^3+70\cdot 5^4+140\cdot 5^3+5^4)$

$=160000=(20)^4$

11. (d): $\cot\left(\displaystyle\sum_{n=1}^{19}\cot^{-1}\left(1+\sum_{p=1}^{n}2p\right)\right)$

$$=\cot\left(\sum_{n=1}^{19}\cot^{-1}\left(1+2\left(\frac{n(n+1)}{2}\right)\right)\right)$$

$$=\cot\left(\sum_{n=1}^{19}\tan^{-1}\frac{1}{1+n(n+1)}\right)$$

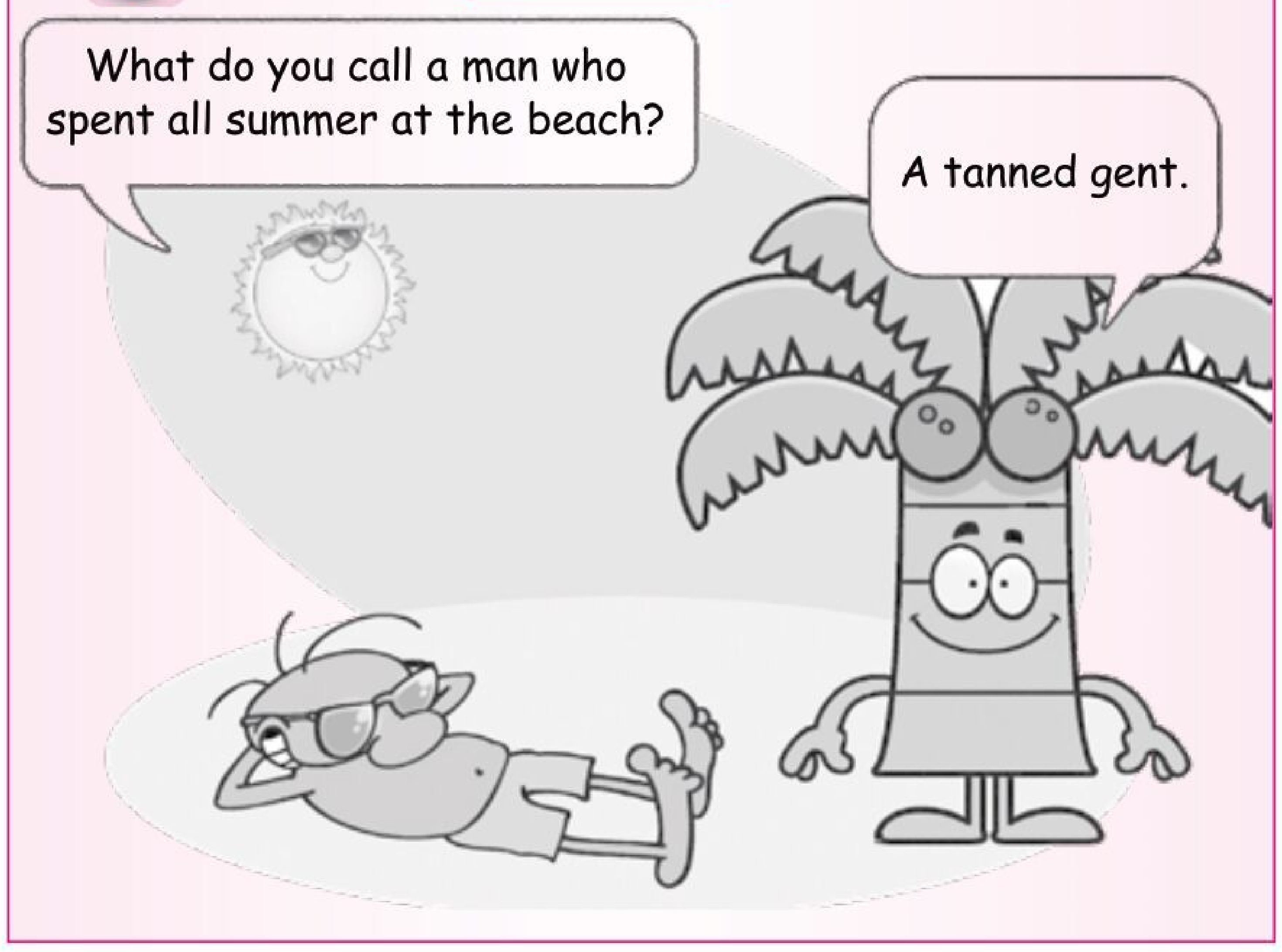
$$=\cot\left(\sum_{n=1}^{19}(\tan^{-1}(n+1)-\tan^{-1}(n))\right)$$

$$\left[\because \tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$$

$=\cot(\tan^{-1}20-\tan^{-1}1)$

$=\dfrac{\cot A\cot B+1}{\cot B-\cot A}$, where $A=\tan^{-1}20$, $B=\tan^{-1}1$

COMIC CAPSULE


What do you call a man who spent all summer at the beach?
A tanned gent.

$$=\frac{\left(\frac{1}{20}\right)(1)+1}{1-\frac{1}{20}}=\frac{21}{19}$$

12. (c) : Given $abc = 2$ and $AA^T = 16I$

Also $A=\begin{pmatrix}3a & b & c\\ b & 3c & a\\ c & a & 3b\end{pmatrix}=A^T$

$\therefore \quad AA^T = 16I \Rightarrow A^2 = 16I$

$\therefore \quad |A|^2=16^3|I| \Rightarrow |A|=\pm 64.\text{But}\,|A|>0\,\text{therefore}\,|A|=64$

Now, $|A|=\begin{vmatrix}3a & b & c\\ b & 3c & a\\ c & a & 3b\end{vmatrix}=64$

$\Rightarrow \quad 29abc - 3(a^3 + b^3 + c^3) = 64$

$\Rightarrow \quad 3(a^3 + b^3 + c^3) = -6 \Rightarrow a^3 + b^3 + c^3 = -2$

$\Rightarrow \quad (a^3 + b^3 + c^3)^2 = 4$

13. (a) : The four diagonals are AL, BM, CN and OP where the co-ordinates of O are $(0, 0, 0)$, $P(a, b, c)$, $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, c)$, $L(0, b, c)$, $M(a, 0, c)$ and $N(a, b, 0)$.

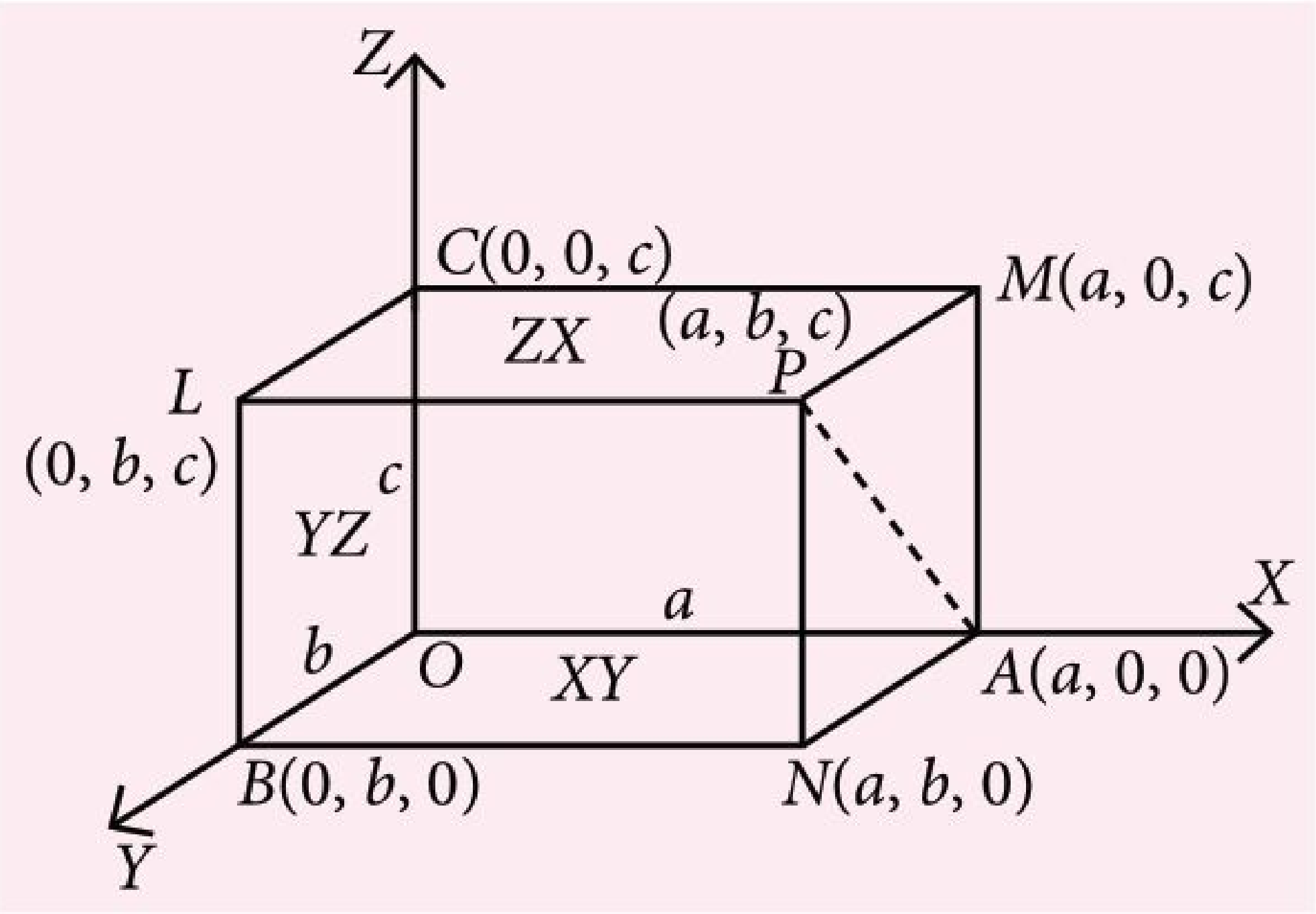
Direction ratios of OP are $a - 0, b - 0, c - 0$. *i.e.*, a, b, c.

$\therefore \quad$ d.c.'s are $\frac{a}{\sqrt{(\Sigma a^2)}}, \frac{b}{\sqrt{(\Sigma a^2)}}, \frac{c}{\sqrt{(\Sigma a^2)}}$

and d.r.'s of LA are

$0 - a, b - 0, c - 0$, *i.e.*, $-a, b, c$

$\therefore \quad$ d.c.'s are $\frac{-a}{\sqrt{(\Sigma a^2)}}, \frac{b}{\sqrt{(\Sigma a^2)}}, \frac{c}{\sqrt{(\Sigma a^2)}}$



Similarly the direction cosines of BM and CN are

$\frac{a}{\sqrt{(\Sigma a^2)}}, \frac{-b}{\sqrt{(\Sigma a^2)}}, \frac{c}{\sqrt{(\Sigma a^2)}}$

and $\frac{a}{\sqrt{(\Sigma a^2)}}, \frac{b}{\sqrt{(\Sigma a^2)}}, \frac{-c}{\sqrt{(\Sigma a^2)}}$ respectively.

If θ be the angle between OP and AL, then

$$\cos\theta=\frac{a(-a)+b(b)+c(c)}{\sqrt{(a^2+b^2+c^2)}\sqrt{(a^2+b^2+c^2)}}$$

$$=\frac{-a^2+b^2+c^2}{a^2+b^2+c^2} \qquad \text{...(i)}$$

Similarly, we can find the angle between other pairs and hence the angles in general between the six pairs are given by

$$\cos^{-1}\left(\frac{\pm a^2 \pm b^2 \pm c^2}{a^2+b^2+c^2}\right)$$

14. (d): Mean $(\bar{x})=\frac{\sum_{i=1}^{50} x_i}{50}=16$ [Given]

$$\Rightarrow \quad \sum_{i=1}^{50} x_i = 16\times 50 \qquad \text{...(i)}$$

Also, standard deviation $(\sigma)=\sqrt{\frac{\sum_{i=1}^{50} x_i^2}{50}-(\bar{x})^2}=16$

$$\Rightarrow \quad \sqrt{\frac{\sum_{i=1}^{50} x_i^2}{50}-(16)^2}=16$$

$$\Rightarrow \quad \frac{\sum_{i=1}^{50} x_i^2}{50}=256\times 2=512 \qquad \text{...(ii)}$$

Now, new mean $=\frac{\sum_{i=1}^{50}(x_i-4)^2}{50}$

$$=\frac{\sum_{i=1}^{50} x_i^2 - 8\sum_{i=1}^{50} x_i + 16\times 50}{50}$$

$= 512 - 8 \times 16 + 16 = 400$ [Using (i) and (ii)]

15. (c) : As $z_n = \text{cis}\left(\frac{\pi}{n(n+1)(n+2)}\right)$; $n = 1, 2, 3, \ldots$

$$\therefore \quad z_1=\text{cis}\left(\frac{\pi}{1\cdot 2\cdot 3}\right), z_2=\text{cis}\frac{\pi}{2\cdot 3\cdot 4},\ldots,z_n=\text{cis}\frac{\pi}{n(n+1)(n+2)}$$

$$\therefore \quad z=\lim_{n\to\infty}(z_1 z_2\ldots z_n)$$

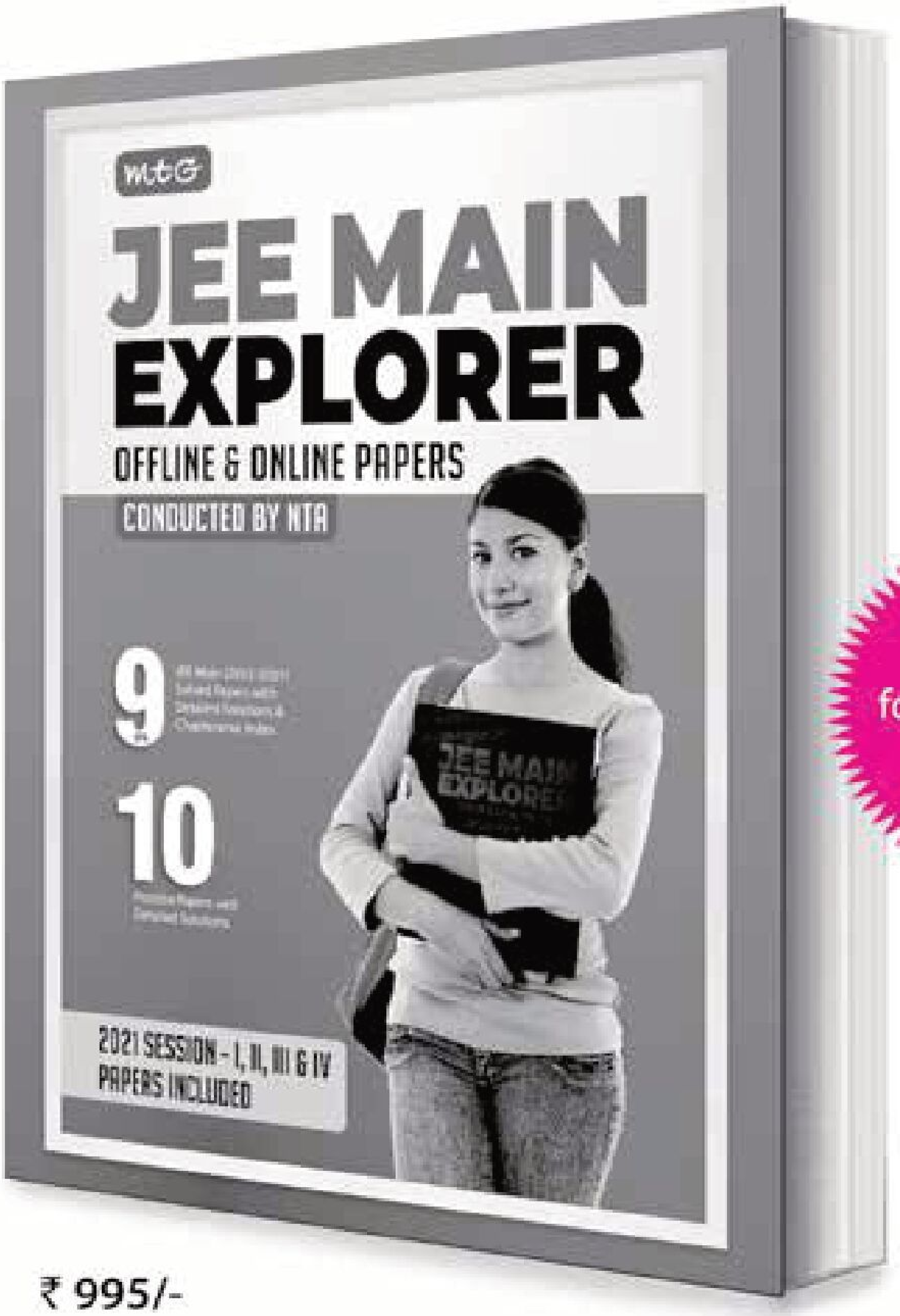
$$=\lim_{n\to\infty}\text{cis}\left(\frac{\pi}{1\cdot2\cdot3}+\frac{\pi}{2\cdot3\cdot4}+\ldots+\frac{\pi}{n(n+1)(n+2)}\right)$$

$$=\lim_{n\to\infty}\text{cis}\left\{\pi\left(\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\ldots+\frac{1}{n(n+1)(n+2)}\right)\right\}$$

$$=\lim_{n\to\infty}\text{cis}\left\{\pi\left(\frac{1}{a_1a_2a_3}+\frac{1}{a_2a_3a_4}+\ldots+\frac{1}{a_na_{n+1}a_{n+2}}\right)\right\}$$

mtG

What is JEE (MAIN) like?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$$= \lim_{n\to\infty} \text{cis}\left\{\pi\left[\frac{1}{2(a_2-a_1)}\left(\frac{1}{a_1a_2}-\frac{1}{a_{n+1}a_{n+2}}\right)\right]\right\}$$

$$= \lim_{n\to\infty} \text{cis}\left\{\frac{\pi}{2}\left(\frac{1}{1\cdot 2}-\frac{1}{(n+1)(n+2)}\right)\right\}$$

$$= \lim_{n\to\infty} \text{cis}\left(\frac{\pi}{2}\cdot\frac{1}{2}-\frac{\pi}{2(n+1)(n+2)}\right)$$

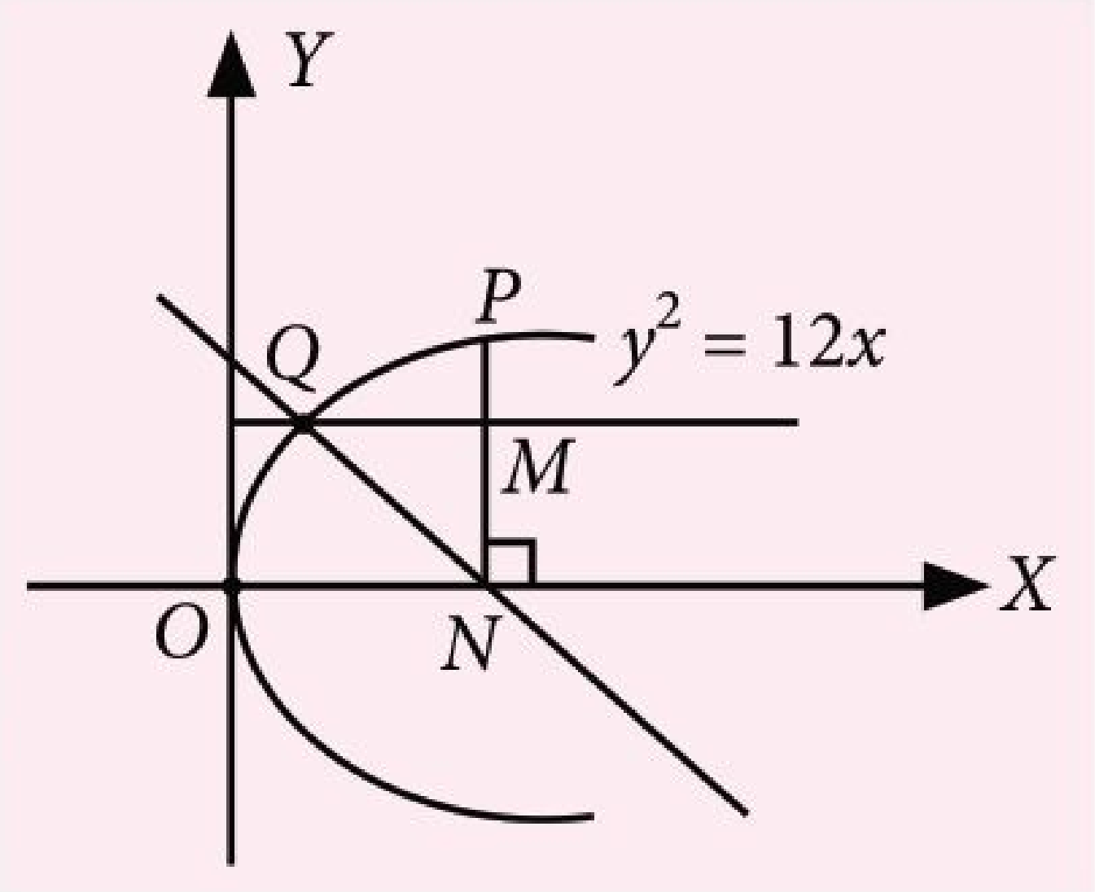
$$= \text{cis}\left(\frac{\pi}{4}-0\right) \qquad \left(\because \frac{\pi}{(n+1)(n+2)} \to 0 \text{ as } n\to\infty\right)$$

$$= \text{cis}\left(\frac{\pi}{4}\right) \Rightarrow \arg(z)=\frac{\pi}{4}$$

16. (c) : We have, $y^2 = 12x$

Let $P(3t^2, 6t)$, $N(3t^2, 0)$, $M(3t^2, 3t)$

Equation of MQ is $y = 3t$



$$\therefore \quad Q = \left(\frac{3}{4}t^2, 3t\right)$$

Equation of NQ is

$$y - 0 = \frac{3t}{\left(\frac{3}{4}t^2 - 3t^2\right)}(x - 3t^2)$$

$$\Rightarrow \quad y = \frac{3t}{3t^2\left(\frac{-3}{4}\right)}(x-3t^2) \Rightarrow y = \frac{-4}{3t}x + 4t$$

Given that y-intercept of $NQ = 4/3$

$$\therefore \quad 4t = \frac{4}{3} \Rightarrow t = \frac{1}{3} \quad \therefore \quad MQ = 3t^2 - \frac{3}{4}t^2 = \frac{9}{4}t^2 = \frac{1}{4}$$

And $PN = 6t = 2$

17. (a) : Let $I = \int \sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{1+x}}\right)dx$

Put $x = \tan^2\theta \Rightarrow \theta = \tan^{-1}\sqrt{x}$

$$\therefore \quad I = \int \sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right)dx = \int \sin^{-1}(\sin\theta)\, dx = \int \theta\, dx$$

$$= \int \tan^{-1}\sqrt{x}\, dx$$

$$= \tan^{-1}\sqrt{x}\cdot\int dx - \int\left(\frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}\cdot\int dx\right)dx$$

$$= \tan^{-1}\sqrt{x}\cdot x - \int \frac{1}{2\sqrt{x}(1+x)}\cdot x\, dx + C$$

$$= x\cdot\tan^{-1}\sqrt{x} - \frac{1}{2}I_1 + C \qquad \text{...(i)}$$

Now, $I_1 = \int \frac{\sqrt{x}}{1+x}dx$

Put $x = t^2 \Rightarrow dx = 2t\, dt$

$$\therefore \quad I_1 = \int \frac{t\cdot 2t\, dt}{1+t^2} = 2\int \frac{1+t^2-1}{1+t^2}\, dt = 2\int\left(1-\frac{1}{1+t^2}\right)dt$$

$$= 2\left[t - \tan^{-1}t\right] = 2\left[\sqrt{x} - \tan^{-1}\sqrt{x}\right]$$

From (i), we have

$$I = x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$$

$$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$$

So, $A(x) = x + 1$, $B(x) = -\sqrt{x}$

18. (b): Series using the last element of the each group is $1^3, 3^3, 6^3, 10^3, ...$

$\therefore$ t_n of 1, 3, 6, 10, ... is $1/2\, n\,(n + 1)$

Now, if we replace n by $n - 1$, we get the last element of $(n - 1)^{\text{th}}$ group

$\therefore$ Last element of the $(n - 1)^{\text{th}}$ group $= 1/2\, n\,(n - 1)$, if we add 1 in it and cubing the number that number be the first element of n^{th} group. So the terms in the n^{th} group are

$$\left(\frac{n(n-1)}{2}+1\right)^3, \left(\frac{n(n-1)}{2}+2\right)^3, \ldots\ldots, \left(\frac{n(n+1)}{2}\right)^3$$

$\therefore$ Sum of the elements of n^{th} group

$= \text{sum of the cubes of first } \frac{n(n+1)}{2} \text{ natural numbers}$

$\quad - \text{sum of the cubes of first } \frac{n(n-1)}{2} \text{ natural numbers}$

$$= \frac{\left(\frac{n(n+1)}{2}\right)^2\left(\frac{n(n+1)}{2}+1\right)^2}{4} - \frac{\left(\frac{n(n-1)}{2}\right)^2\left(\frac{n(n-1)}{2}+1\right)^2}{4}$$

$$= \frac{n^3}{8}(n^2+1)(n^2+3) \qquad \text{(On simplification)}$$

19. (a) : Given, $\frac{dy}{dx} = \frac{mx+5}{4y+f}$

$\Rightarrow \quad (4y + f)dy = (mx + 5)dx$

$$\Rightarrow \quad \frac{4y^2}{2} + fy = \frac{mx^2}{2} + 5x + k$$

$$\Rightarrow \quad 2y^2 - \frac{mx^2}{2} + fy - 5x - k = 0 \qquad \text{...(*)}$$

will represent the circle if $-\frac{m}{2} = 2 \Rightarrow m = -4$

Again from (*) we have

$2(x^2 + y^2) + fy - 5x - k = 0$

$$\therefore \quad x^2 + y^2 + 2\left(\frac{f}{4}\right)y + 2\left(\frac{-5}{4}\right)x - \frac{k}{2} = 0$$

$$\therefore \quad r = \sqrt{\frac{f^2}{16} + \frac{25}{16} + \frac{k}{2}} = \frac{1}{4}\sqrt{f^2 + 8k + 25}$$

$$\therefore \quad m = -4 \;\&\; r = \frac{1}{4}\sqrt{f^2 + 8k + 100}$$

20. (b): Given mean $= 91 \times 2^{3-n}$

x_i	f_i	$x_i f_i$
0	${}^nC_0$	0
2^1	${}^nC_1$	$2 \cdot {}^nC_1$
2^2	${}^nC_2$	$2^2 \cdot {}^nC_2$
2^3	${}^nC_3$	$2^3 \cdot {}^nC_3$
.	.	.
.	.	.
.	.	.
2^n	${}^nC_n$	$2^n \cdot {}^nC_n$
	$\Sigma f_i = 2^n$	$\Sigma f_i x_i = 3^n - 1$

Now mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow \dfrac{91 \times 2^3}{2^n} = \dfrac{3^n - 1}{2^n}$

$\Rightarrow \quad 3^n = 3^6 \Rightarrow n = 6$

21. (238) : There are 5 students in class 10, 6 students in class 11 and 8 students in class 12.

Number of ways, in which 10 students can be selected, according to given conditions

10th class	11th class	12th class	
2	2	6	$\Rightarrow {}^5C_2 \times {}^6C_2 \times {}^8C_6$
2	3	5	$\Rightarrow {}^5C_2 \times {}^6C_3 \times {}^8C_5$
3	2	5	$\Rightarrow {}^5C_3 \times {}^6C_2 \times {}^8C_5$

$\therefore$ Total number of ways $= {}^5C_2 \times {}^6C_2 \times {}^8C_6 + {}^5C_2 \times {}^6C_3 \times {}^8C_5 + {}^5C_3 \times {}^6C_2 \times {}^8C_5 = 23800$

But total number of ways $= 100k$ [Given]

$\Rightarrow \quad 100k = 23800 \;\Rightarrow\; k = 238$

22. (2011) : On comparing with $ax^2 + 2hxy + by^2 = 0$, we have $a = 11, 2h = -100, b = 5$

If m_1 and m_2 be slopes of lines represented by $11x^2 - 100xy + 5y^2 = 0$

then $s = m_1 + m_2 = -\dfrac{2h}{b} = 20;\; p = m_1 m_2 = \dfrac{a}{b} = \dfrac{11}{5}$

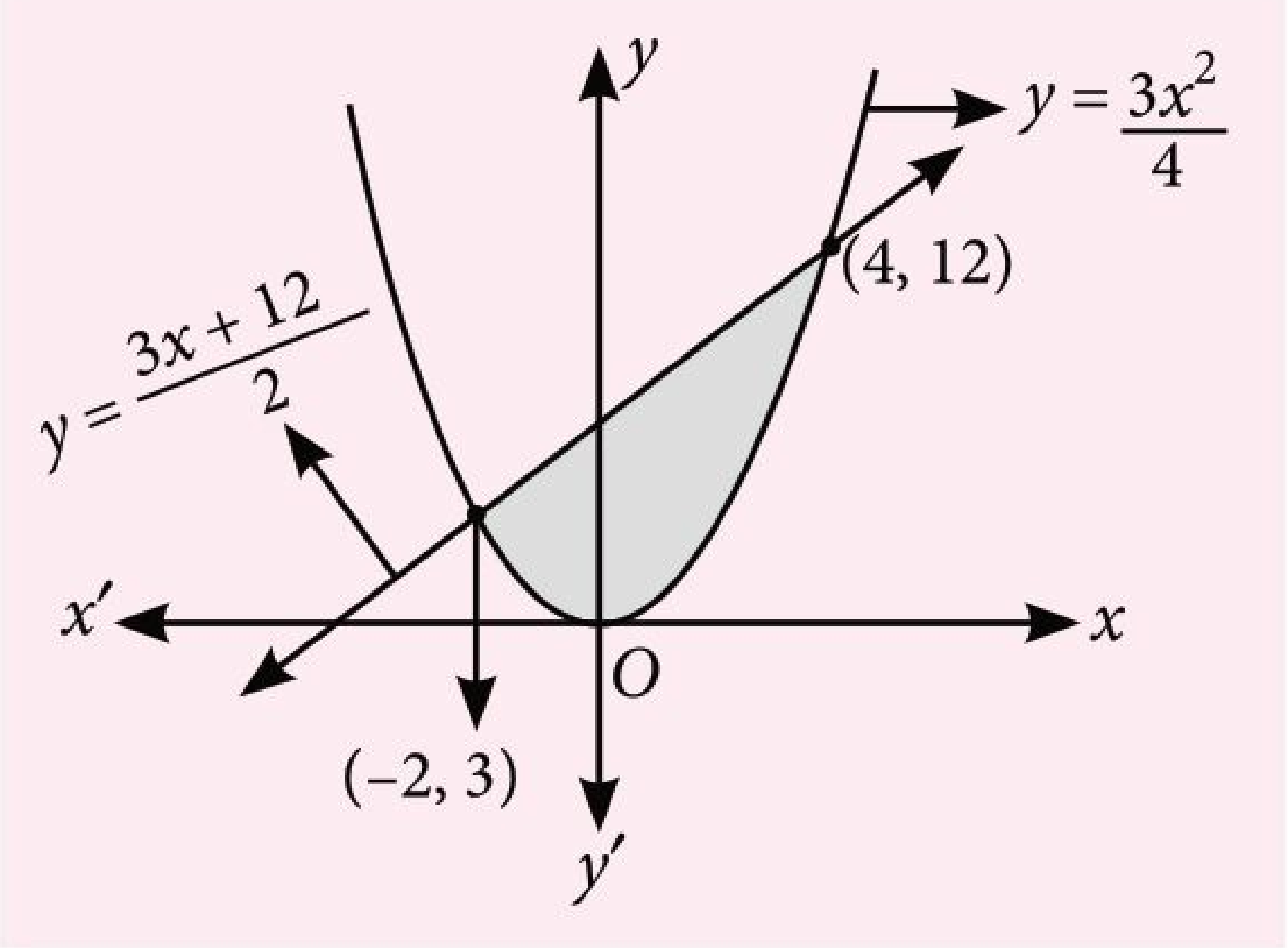
$$\therefore \quad 5(p + 20s) = 5\left(\frac{11}{5} + 400\right) = 2011$$

23. (27) : We have, $\dfrac{3x^2}{4} \le y \le \dfrac{3x + 12}{2}$

For point of intersection, we have $3x^2 = 6x + 24$

$\Rightarrow \quad x^2 - 2x - 8 = 0 \;\Rightarrow\; (x-4)(x+2) = 0$

$\Rightarrow \quad x = 4$ and $x = -2$



$$\therefore \quad \text{Required area} = \int_{-2}^{4} \left(\frac{3x + 12}{2} - \frac{3x^2}{4} \right) dx$$

$$= \left[\frac{3x^2}{4} + 6x - \frac{x^3}{4} \right]_{-2}^{4}$$

$= (12 + 24 - 16) - (3 - 12 + 2) = 20 - (-7) = 27$

24. (1) : For circle, $x^2 + y^2 - 10x - 10y + 41 = 0$

Centre $C_1 \equiv (5, 5)$ and radius, $r_1 = 3$

For circle, $x^2 + y^2 - 24x - 10y + 160 = 0$

Centre $C_2 \equiv (12, 5)$ and radius, $r_2 = 3$

$C_1C_2 = 7 > r_1 + r_2 \Rightarrow$ Circles are separated

$\therefore$ Required minimum possible distance, P_1P_2

$= 7 - (3 + 3) = 1$

25. (45) : We have, $z_0 = \omega$ or ω^2

(Where ω is a non real cube root of unity)

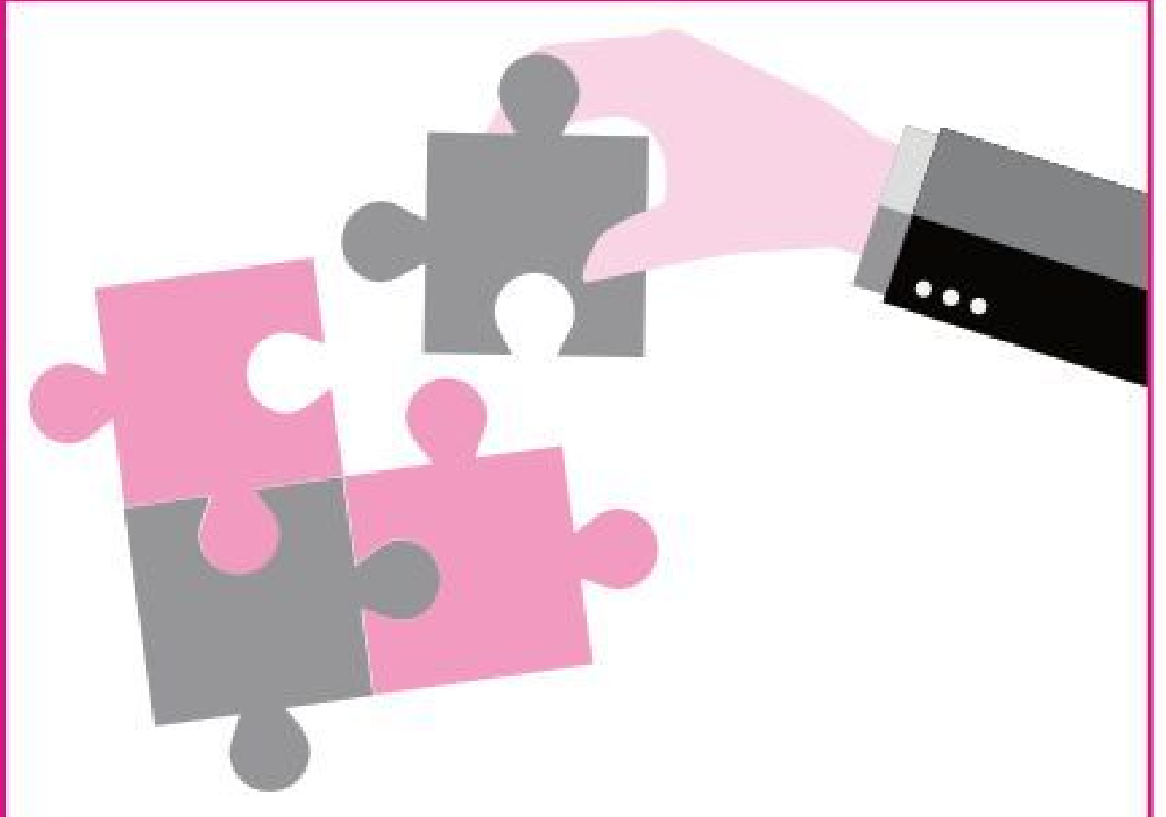
If $z_0 = \omega$, then $z = 3 + 6iz_0^{81} - 3iz_0^{93}$

$= 3 + 6i(\omega)^{81} - 3i(\omega)^{93} = 3 + 6i - 3i = 3 + 3i$

If $z_0 = \omega^2$, then $z = 3 + 6iz_0^{81} - 3iz_0^{93}$

$= 3 + 6i(\omega^2)^{81} - 3i(\omega^2)^{93} = 3 + 6i(\omega)^{162} - 3i(\omega)^{186}$

PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

3	1−	6×	4−		1−
1−			3+	3	
	1			18+	
15×			20×		
3÷	32×	6		15×	4+

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' names with their valuable feedback will be published in next issue.

$= 3 + 6i - 3i = 3 + 3i$

$\therefore \quad \arg(z) = \tan^{-1}\left(\frac{3}{3}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

26. (5) : We have,

$$f(x) = \begin{cases} x^5 \sin\left(\frac{1}{x}\right) + 5x^2, & x < 0 \\ 0, & x = 0 \\ x^5 \cos\left(\frac{1}{x}\right) + \lambda x^2, & x > 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 5x^4 \sin\left(\frac{1}{x}\right) + x^5 \cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) + 10x, & x<0 \\ 0, & x=0 \\ 5x^4 \cos\left(\frac{1}{x}\right) + x^5\left(-\sin\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)\right) + 2\lambda x, & x>0 \end{cases}$$

$$\Rightarrow \quad f'(x) = \begin{cases} 5x^4 \sin\left(\frac{1}{x}\right) - x^3 \cos\left(\frac{1}{x}\right) + 10x, & x < 0 \\ 0, & x = 0 \\ 5x^4 \cos\left(\frac{1}{x}\right) + x^3 \sin\left(\frac{1}{x}\right) + 2\lambda x, & x > 0 \end{cases}$$

$$\Rightarrow \quad f''(x) = \begin{cases} [20x^3 \sin\left(\frac{1}{x}\right) - 5x^2 \cos\left(\frac{1}{x}\right) \\ -3x^2 \cos\left(\frac{1}{x}\right) - x \sin\left(\frac{1}{x}\right) + 10], & x < 0 \\ 0, & x = 0 \\ [20x^3 \cos\left(\frac{1}{x}\right) + 5x^2 \sin\left(\frac{1}{x}\right) \\ + 3x^2 \sin\left(\frac{1}{x}\right) - x \cos\left(\frac{1}{x}\right) + 2\lambda], & x > 0 \end{cases}$$

To exist $f''(0), f''(0^+) = f''(0^-)$

$\Rightarrow \quad 2\lambda = 10 \quad \Rightarrow \quad \lambda = 5$

27. (9) : If the line cuts the circle, then

$CP < r.$ ...(i)

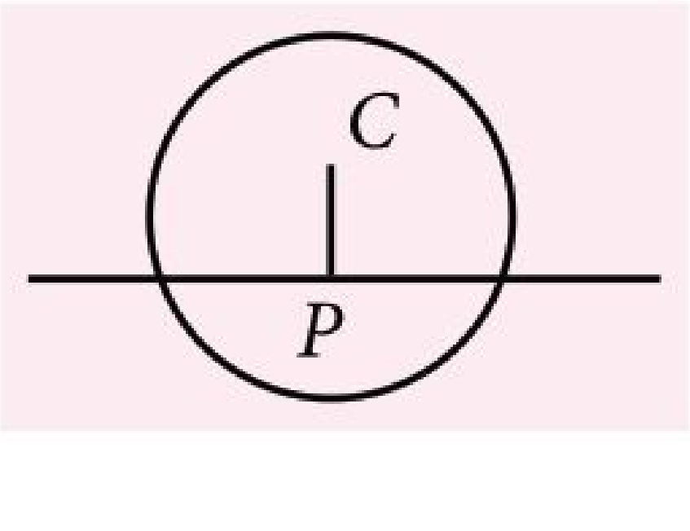
Since, equation of circle is $x^2 + y^2 - 2x - 4y + 4 = 0$

$\therefore$ Centre = (1, 2) and radius = $\sqrt{1 + 4 - 4} = 1$

Also, the equation of line is $3x + 4y = k$.

$\therefore$ Using (i), $\left|\frac{3 + 8 - k}{\sqrt{9 + 16}}\right| < 1$

$\Rightarrow \left|\frac{11 - k}{5}\right| < 1 \Rightarrow |11 - k| < 5$



$\Rightarrow \quad 11 - k < 5$ and $-(11 - k) < 5$

$\Rightarrow \quad k > 6$ and $-11 + k < 5 \Rightarrow k < 16 \Rightarrow k \in (6, 16)$

Integral values of k = 7, 8, 9, 10, 11, 12, 13, 14, 15

Thus, number of integral values of k is 9.

28. (1728) : Let $z = a + b\omega + c\omega^2$

$\Rightarrow \quad \bar{z} = \overline{a + b\omega + c\omega^2} = a + b\omega^2 + c\omega$

Now, $|z|^2 = z\bar{z} = (a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$

$= a^2 + b^2 + c^2 + ab(\omega + \omega^2) + bc(\omega + \omega^2) + ca(\omega + \omega^2)$

$= a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$

$\therefore \quad |z| = \frac{1}{\sqrt{2}}\sqrt{(a-b)^2 + (b-c)^2 + (c-a)^2}$

Again $|a + b\omega + c\omega^2| + |a + b\omega^2 + c\omega| = |z| + |\bar{z}| = 2|z|$

$\therefore \quad |a + b\omega + c\omega^2| + |a + b\omega^2 + c\omega|$

$= \sqrt{2}\sqrt{(a-b)^2 + (b-c)^2 + (c-a)^2} \geq \sqrt{2} \times \sqrt{(-1)^2 + (-1)^2 + 2^2}$

($\because \quad b = a + 1, c = b + 1$ which means a, b, c are consecutive)

$= (12)^{1/2} = (12^3)^{1/6} = (1728)^{1/6} = \alpha^{1/6}$ (Given)

$\therefore \quad \alpha = 1728$

29. (2454) : The word EXAMINATION has

2N, 2A, 2I, E, X, M, T, O

Case I : If all letters are different, then number of words

$= {}^8P_4 = \frac{8!}{4!} = 8 \cdot 7 \cdot 6 \cdot 5 = 1680$

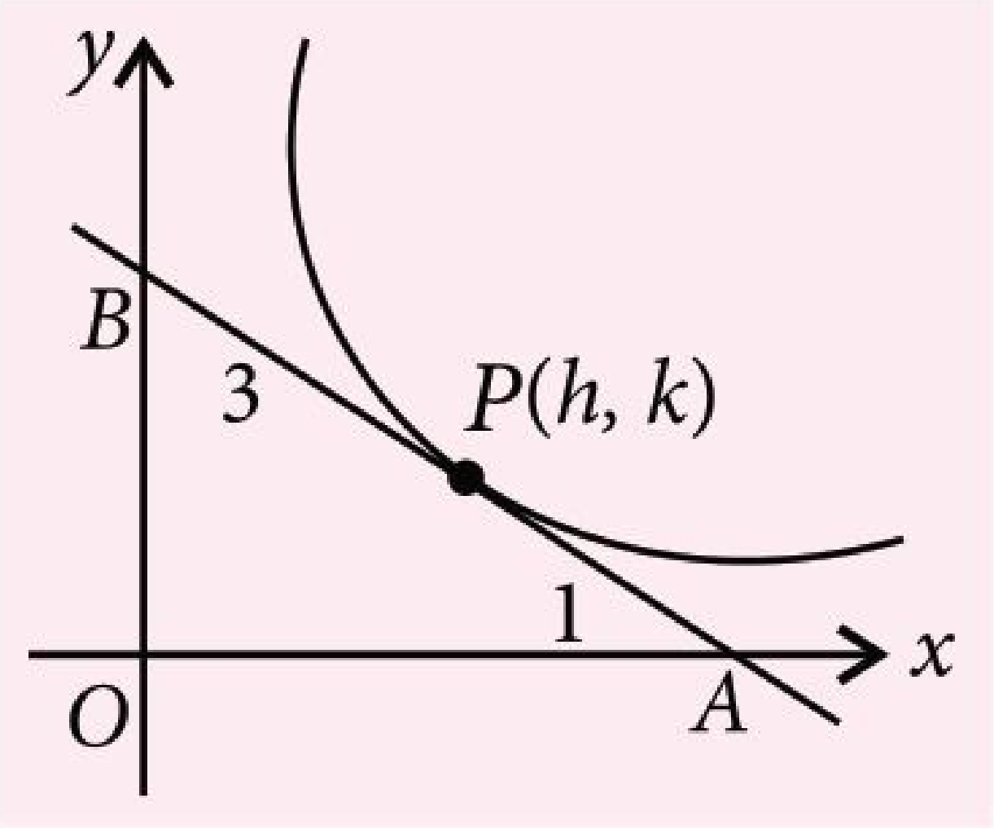
Case II : If 2 letters are same and 2 are different, then number of words = ${}^3C_1 \cdot {}^7C_2 \cdot \frac{4!}{2!} = 3 \cdot 21 \cdot 12 = 756$

Case III : If 2 letters are same of one kind and 2 letters are same of other kind, then number of words

$= {}^3C_2 \cdot \frac{4!}{2!\,2!} = 3 \cdot 6 = 18$

$\therefore$ Total number of words = 1680 + 756 + 18 = 2454

30. (2) : $\frac{3x}{h} + \frac{y}{k} = 1$ is the equation of the tangent whose slope is $-\frac{3k}{h}$.



$\frac{dy}{dx} = -\frac{3y}{x} \Rightarrow x\frac{dy}{dx} + 3y = 0$

$\Rightarrow \quad \frac{dy}{y} = -3\frac{dx}{x} \Rightarrow \log y = -3\log x + \log c$

$\Rightarrow \quad y = cx^{-3} \Rightarrow y = \frac{c}{x^3}$, which passes through (1,1)

$\Rightarrow \quad c = 1$

$\therefore \quad$ Equation of the curve is $y = \frac{1}{x^3}$.

$\because \quad$ It passes through $\left(k, \frac{1}{8}\right)$

$\therefore \quad \frac{1}{8} = \frac{1}{k^3} \Rightarrow k = 2$

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BRUSH UP for JEE Main & Advanced 2022

Brush up your concepts to get high rank in JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year according to the requirements of these Entrance Tests.

Single Correct Answer Type

1. Which of the following function is injective map?

(a) $f(x)=x^2+2,\ x\in(-\infty,\ \infty)$

(b) $f(x)=|x+2|,\ x\in[-2,\ \infty)$

(c) $f(x)=(x-4)(x-5),\ x\in(-\infty,\ \infty)$

(d) $f(x)=\dfrac{4x^2+3x-5}{4+3x-5x^2},\ x\in(-\infty,\ \infty)$

2. Let $f(x)=\dfrac{1}{\sqrt{|x-1|-[x]}}$ where $[\cdot]$ denote the greatest integer function, then domain of $f(x)$ is

(a) $(-1,\ 1)$ (b) $(-\infty,\ 1)$

(c) $(-\infty,\ -1)$ (d) none of these

3. If $x_1, x_2, x_3, \ldots, x_n$ are the roots of the equation, $x^n + ax + b = 0$, then the value of $(x_1 - x_2)(x_1 - x_3)(x_1 - x_4) \ldots (x_1 - x_n)$ is equal to

(a) $nx_1^{n-1} + a$ (b) $n(x_1)^{n-1}$

(c) $nx_1 + b$ (d) $nx_1^{n-1} + b$

4. Matrix A is such that $A^2 = 2A - I$, where I is unit matrix, then for $N \geq 2$, A^n is equal to

(a) $nA - (n-1)I$ (b) $nA - I$

(c) $2^{n-1}A - (n-1)I$ (d) $2^{n-1}A - I$

5. Foot of perpendicular drawn from origin to the plane passing through (1, 0, 0), (0, 1, 0) and (0, 0, 1) is

(a) (3, 3, 3) (b) $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

(c) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (d) (2, 2, 2)

6. $\vec{a}, \vec{b}, \vec{c}$ are three vectors so that each one is perpendicular to the sum of the other two and also $|\vec{a}|=5, |\vec{b}|=6, |\vec{c}|=7$. Then $\left|\vec{a}+\vec{b}+\vec{c}\right|$ is

(a) $2\sqrt{55}$ (b) $\sqrt{110}$

(c) $\sqrt{55}$ (d) none of these

7. The condition to the imposed on β so that $(0, \beta)$ lies on or inside the triangle having sides $y = 3x + 2 = 0$, $3y - 2x - 5 = 0$ and $4y + x - 14 = 0$ is

(a) $0<\beta<\frac{5}{3}$ (b) $0<\beta<\frac{7}{2}$

(c) $\frac{5}{3}\leq\beta\leq\frac{7}{2}$ (d) $\frac{5}{2}\leq\beta\leq\frac{7}{2}$

8. If $a, b, c, d > 0$; $x \in R$ and $(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \leq 0$, then $\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix}$ is equal to

(a) 1 (b) –1

(c) 0 (d) 2

9. $\displaystyle\int_a^b f(x)\,dx$ is equal to

(a) $\displaystyle\int_{a+t}^{b+t} f(x)\,dx$ (b) $\displaystyle\int_{a-t}^{b-t} f(x-t)\,dx$

(c) $\displaystyle\int_{a-t}^{b-t} f(x+t)\,dx$ (d) none of these

10. The mean deviation of an ungrouped data is 10. If each observation is increased by 4%, then the revised mean deviation is

(a) 10.4 (b) 10.04
(c) 9.6 (d) 10.0

11. If A and B are square matrices of the same order, then which of the following is always true?
(a) $\text{adj}(AB) = (\text{adj}B)(\text{adj}A)$
(b) A and B are non-zero and $|AB| = 0 \Leftrightarrow |A| = 0$ and $|B| = 0$
(c) $(AB)^{-1} = A^{-1}B^{-1}$
(d) $(A + B)^{-1} = A^{-1} + B^{-1}$

12. Two variable chords AB and BC of a circle $x^2 + y^2 = r^2$ are such that $AB = BC = r$. Then the locus of point of intersection of tangents at A and C is
(a) $x^2 + y^2 = 2r^2$ (b) $x^2 + y^2 = 3r^2$
(c) $x^2 + y^2 = 4r^2$ (d) $x^2 + y^2 = 5r^2$

13. Consider the equation $x^4 + x^2 + 1 = 0$. If x_1, x_2, x_3, x_4 are roots of this equation, then value of $x_1^6 + x_2^6 + x_3^6 + x_4^6$ equals
(a) 1 (b) 2 (c) 3 (d) 4

14. If z lies on the circle $|z - 1| = 1$, then $\dfrac{z-2}{z}$ equals
(a) 0 (b) 2
(c) −1 (d) none of these

15. One hundred identical coins, each with probability p of showing heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, the value of p is
(a) $\dfrac{1}{2}$ (b) $\dfrac{51}{101}$
(c) $\dfrac{49}{101}$ (d) none of these

16. On the portion of the straight line $x + y = 12$ which is intercepted between the axes, a square is constructed, away from the origin, with this portion as one of its side. If p denotes the perpendicular distance of a side of this square from the origin, then the maximum value of p is
(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $\dfrac{2}{\sqrt{3}}$ (d) $\dfrac{3}{\sqrt{2}}$

17. If P is a point on the rectangular hyperbola $x^2 - y^2 = a^2$, C is its centre and S, S' are the two foci, then $SP{\cdot}S'P$ is equal to
(a) 2 (b) $(CP)^2$
(c) $(CS)^2$ (d) $(SS')^2$

18. The angle between the lines whose direction cosines are given by the equations $l^2 + m^2 - n^2 = 0$, $l + m + n = 0$ is
(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

19. Let $f(x) = \dfrac{|x|}{x}$ if $x \neq 0$ and $f(0) = 0$ and $a, b \in R$ be such that $a < b$, then value of $I = \int_a^b f(x)\,dx$ is
(a) $|b| - |a|$ (b) $\dfrac{1}{2}(b^2 - a^2)$
(c) $\max\{|a|, |b|\}$ (d) $\min\{|a|, |b|\}$

20. If ω is complex cube root of unity, and $A = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then A^{100} is equal to
(a) A (b) $-A$
(c) O (d) none of these

21. The value of the determinant $\begin{vmatrix} ka & k^2+a^2 & 1 \\ kb & k^2+b^2 & 1 \\ kc & k^2+c^2 & 1 \end{vmatrix}$ is
(a) $k(a+b)(b+c)(c+a)$
(b) $kabc(a^2+b^2+c^2)$
(c) $k(a-b)(b-c)(c-a)$
(d) $k(a+b-c)(b+c-a)(c+a-b)$

22. If in the expansion of $\left(x^3 - \dfrac{1}{x^2}\right)^n, n \in N$, sum of the coefficient of x^5 and x^{10} is 0, then value of n is
(a) 5 (b) 10
(c) 15 (d) none of these

23. If A, B and C are three events such that $P(B) = \dfrac{3}{4}$, $P(A \cap B \cap C') = \dfrac{1}{3}$ and $P(A' \cap B \cap C') = \dfrac{1}{3}$, then $P(B \cap C)$ is equal to
(a) $\dfrac{1}{12}$ (b) $\dfrac{1}{6}$ (c) $\dfrac{1}{15}$ (d) $\dfrac{1}{9}$

24. The value of $\sin\left(2\tan^{-1}\left(\dfrac{1}{3}\right)\right) + \cos(\tan^{-1} 2\sqrt{2})$ is
(a) $\dfrac{12}{13}$ (b) $\dfrac{13}{14}$
(c) $\dfrac{14}{15}$ (d) none of these

25. If the lines $x + 3y = 4$ and $3x + y = 4$ cuts the coordinate axes at four concyclic points, then the radius of that circle is equal to
(a) $\dfrac{4\sqrt{5}}{3}$ (b) $\dfrac{3\sqrt{5}}{4}$ (c) $\dfrac{4}{3}$ (d) $\dfrac{\sqrt{5}}{3}$

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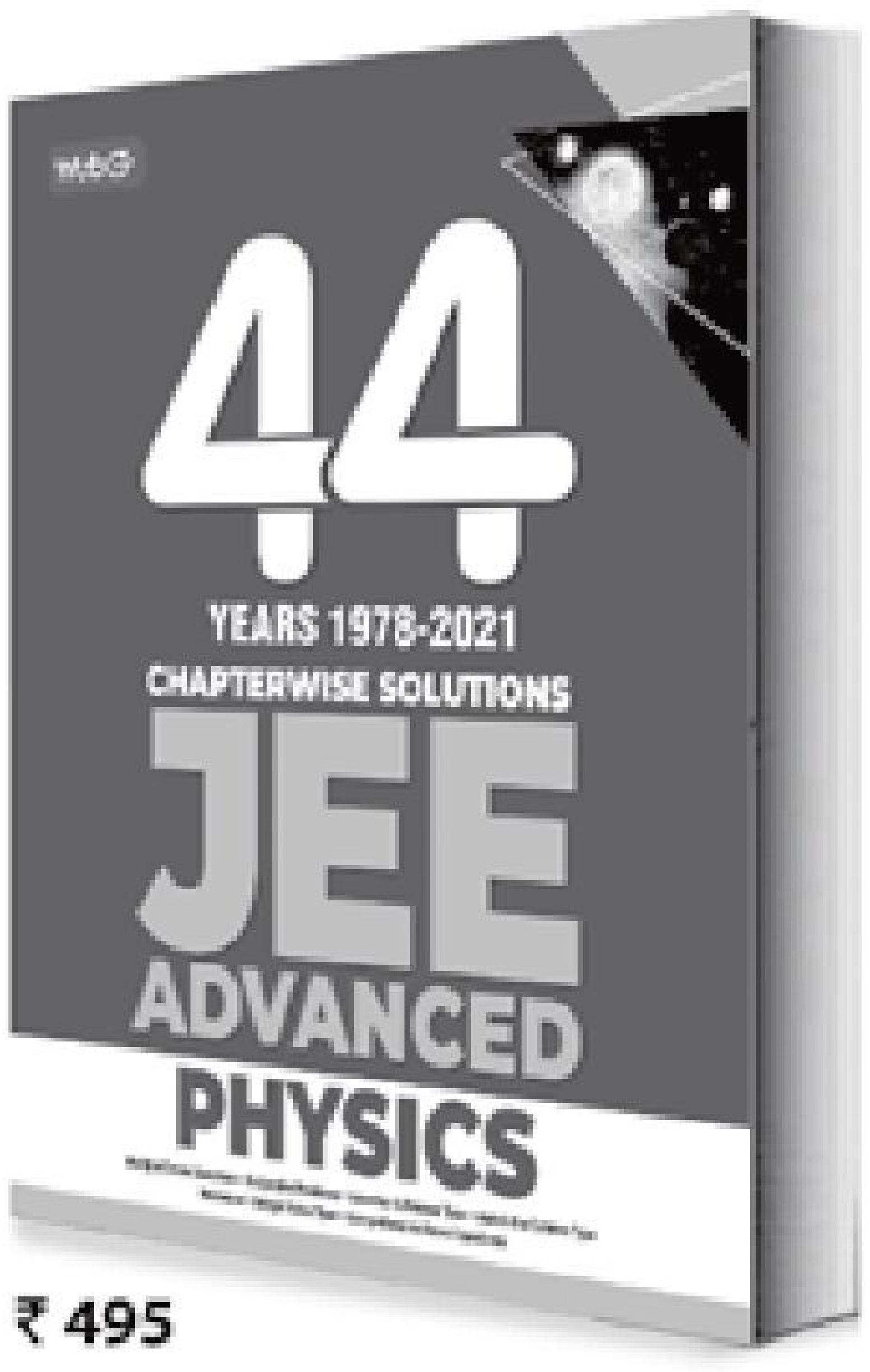
How can history help to succeed in JEE!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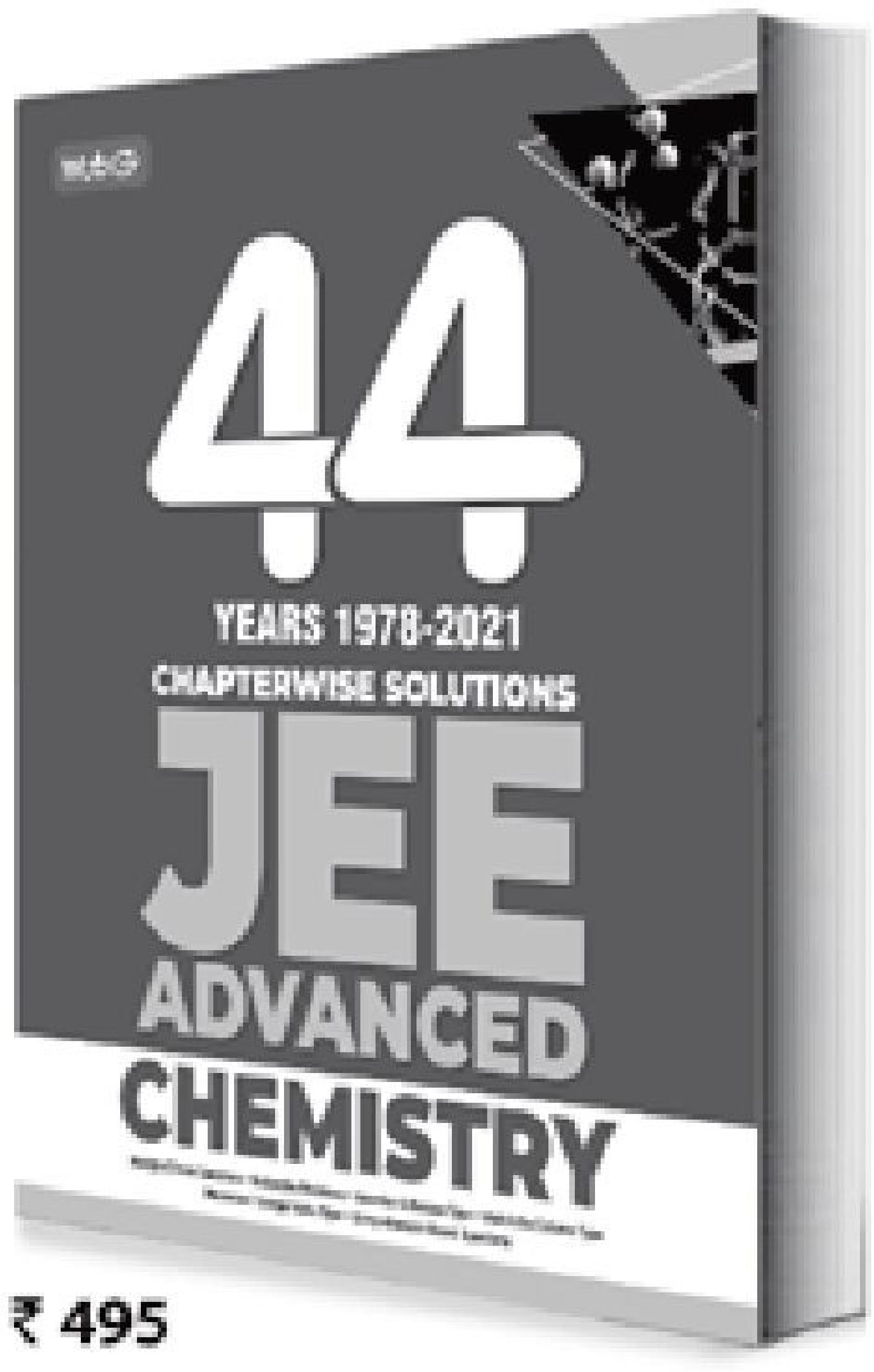
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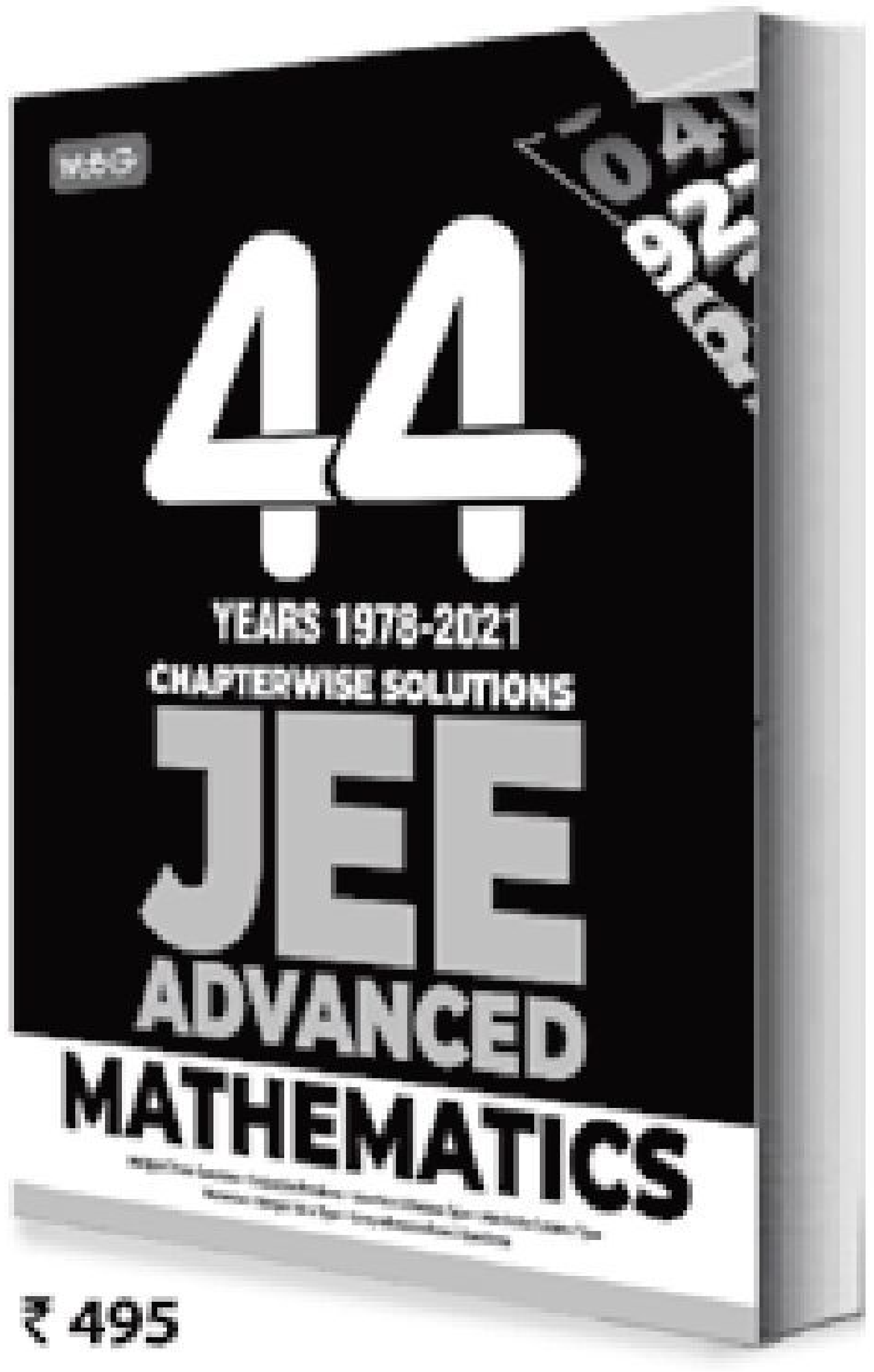
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26. If the coordinates axes are rotated by an angle 45° in clockwise direction (keeping origin fixed), then equation of hyperbola $x^2 - y^2 = 2$ becomes

(a) $xy - 1 = 0$ (b) $xy + 1 = 0$

(c) $xy + 2 = 0$ (d) none of these

Multiple Correct Answer Type

27. Orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2 = \frac{a}{x}$ are

(a) $9a(y + c)^2 = 4x^3$ (b) $y + c = \frac{-2}{3\sqrt{a}}x^{3/2}$

(c) $y + c = \frac{2}{3\sqrt{a}}x^{3/2}$ (d) $9a(y - c)^2 = 5x^3$

28. If $\frac{{}^nC_1}{{}^nC_0} + 2\frac{{}^nC_2}{{}^nC_1} + 3\frac{{}^nC_3}{{}^nC_2} + ... + n\frac{{}^nC_n}{{}^nC_{n-1}} = S$, then S will be

(a) an integer (b) $\frac{1}{2}n(n-1)$

(c) $\frac{1}{2}n(n+1)$ (d) 55, if $n = 10$

29. If a, b, c, d, e are positive real numbers such that $a + b + c + d + e = 8$ and $a^2 + b^2 + c^2 + d^2 + e^2 = 16$, then

(a) $1 \le e < 2$ (b) $0 < e \le \frac{16}{5}$

(c) $2 < e < 3$ (d) $0 < e < 4$

30. $\sin\theta + \sqrt{3}\cos\theta = 6x - x^2 - 11,\ 0 \le \theta \le 4\pi,\ x \in R$ holds for

(a) no value of x and θ

(b) one value of x and two values of θ

(c) two values of x and two values of θ

(d) two pair of values of (x, θ)

Comprehension Type

Paragraph for Q.No. 31 to 33 : If a, b are prime numbers, $n \in N$, then the rational terms in the expansion of $(a^{1/p} + b^{1/q})^n$ are the terms in which indices of a and b are integral numbers. On the basis of the above information answer the following questions.

31. In the expansion of $(\sqrt{2} + \sqrt[5]{3})^{120}$ the number of irrational terms is

(a) 12 (b) 13 (c) 108 (d) 54

32. In the expansion $(2^{1/5} + 7^{1/7})^{105}$ the sum of rational terms is the sum of the terms

(a) $T_1 + T_{35} + T_{70} + T_{105}$

(b) $T_1 + T_{36} + T_{71} + T_{106}$

(c) $T_1 + T_{34} + T_{69} + T_{104}$

(d) none of these

33. In the expansion of $(7^{1/3} + 11^{1/9})^{5832}$, the number of terms free from radicals is

(a) 649 (b) 648 (c) 72 (d) 647

Paragraph for Q.No. 34 to 36 : If a source of light is placed at the fixed point of a parabola and if the parabola is a reflecting surface, then the ray bounce back in a line parallel to the axis of the parabola.

34. A ray of light is coming along the line $y = 4$ from the positive direction of x-axis and strikes a concave mirror whose intersection with the plane XOY is a parabola $y^2 = 16x$, then equation of reflected ray is

(a) $3x + 2y = 16$ (b) $4x + 3y = 16$

(c) $5x + 4y = 9$ (d) none of these

35. A ray of light moving parallel to x-axis gets reflected from the parabolic mirror whose equation is given by $y^2 - 10y - 8x + 41 = 0$, after reflection, the ray must passes through the point

(a) $(4, 5)$ (b) $(-4, -5)$

(c) $(-4, 5)$ (d) $(4, -5)$

36. A ray of light is coming along the line $x = 2$ from positive direction of y-axis and strikes a concave mirror whose intersection with XOY plane is a parabola $x^2 = 4y$, then equation of reflected ray after the second reflection is

(a) $y + 2 = 0$ (b) $y + 1 = 0$

(c) $x + 2 = 0$ (d) none of these

SOLUTIONS

1. **(b)** **2.** **(b)**

3. **(a) :** $x^n + ax + b = (x - x_1)(x - x_2) \ldots. (x - x_n)$

$$\Rightarrow (x - x_2)(x - x_3)........(x - x_n) = \frac{x^n + ax + b}{x - x_1}$$

$$= \lim_{x \to x_1} \frac{x^n + ax + b}{x - x_1} = nx_1^{n-1} + a$$

4. **(a) :** $A^2 = 2A - I$ …(i)

Multiplying by A, we have,

$A^3 = 2A^2 - A = 2(2A - I) - A \Rightarrow A^3 = 3A - 2I$

Again multiplying by A, $A^4 = 3A^2 - 2AI$

$\Rightarrow A^4 = 3(2A - I) - 2A \Rightarrow A^4 = 6A - 3I - 2A$

$\Rightarrow A^4 = 4A - 3I$

Hence by induction $A^n = nA - (n - 1)I$

5. **(c)**

6. (b) : $\vec{a}\cdot(\vec{b}+\vec{c})+\vec{b}\cdot(\vec{c}+\vec{a})+\vec{c}\cdot(\vec{a}+\vec{b})=0$

$$\Rightarrow\ \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=0$$

$$|\vec{a}+\vec{b}+\vec{c}|^2=\sum\vec{a}^2+2\sum\vec{a}\cdot\vec{b}=110$$

7. (c)

8. (c) : $(a^2+b^2+c^2)x^2-2(ab+bc+cd)x+b^2+c^2+d^2\le 0$

$\Rightarrow (ax-b)^2+(bx-c)^2+(cx-d)^2\le 0$

$\Rightarrow (ax-b)^2+(bx-c)^2+(cx-d)^2=0$

$$\Rightarrow\ \frac{b}{a}=\frac{c}{b}=\frac{d}{c}=x\ \Rightarrow b^2=ac$$

or, $2\log b=\log a+\log c$

$$\text{Now, }\begin{vmatrix}33 & 14 & \log a\\ 65 & 27 & \log b\\ 97 & 40 & \log c\end{vmatrix}=\begin{vmatrix}130 & 54 & \log a+\log c\\ 65 & 27 & \log b\\ 97 & 40 & \log c\end{vmatrix}$$

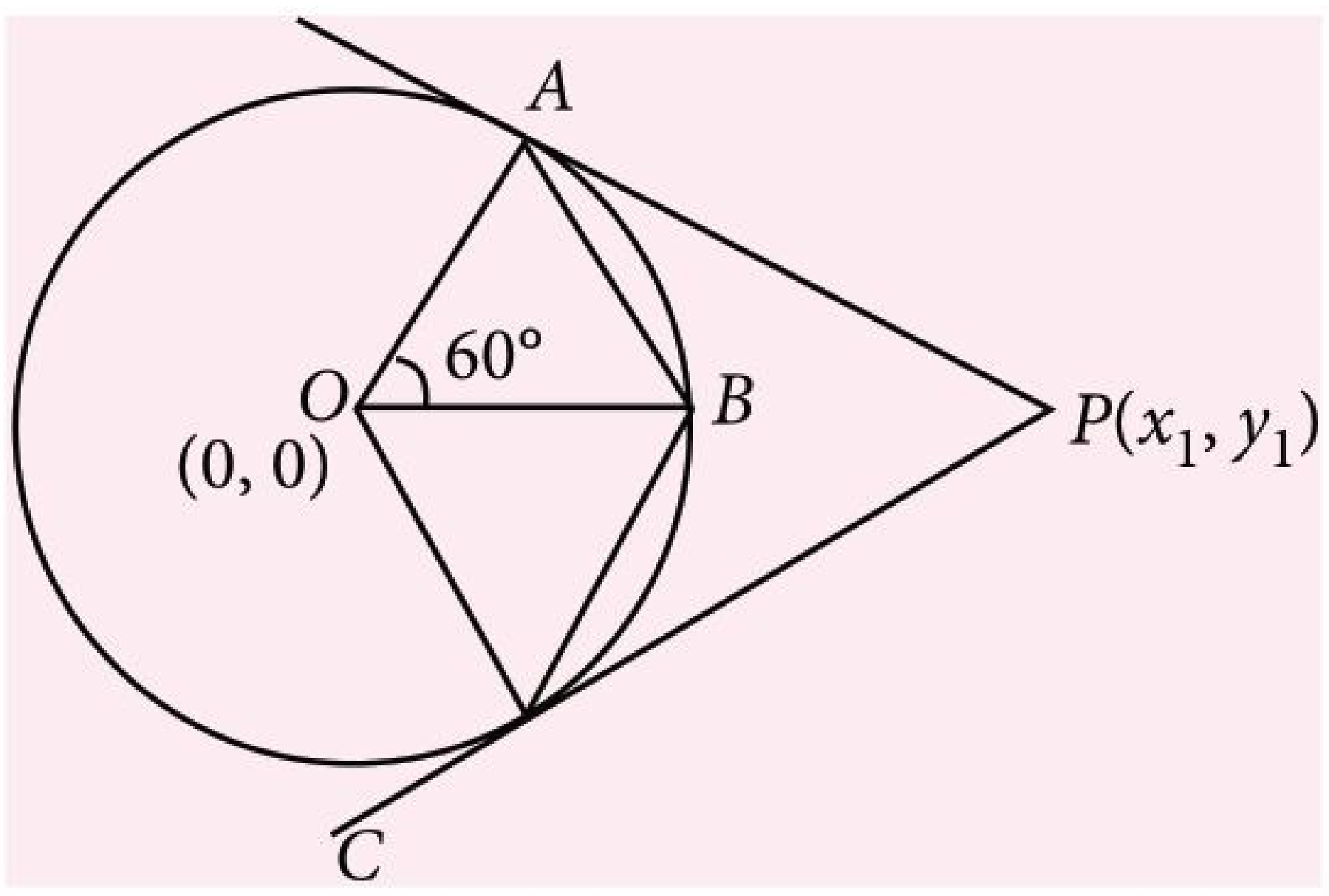
Applying $R_1\to R_1+R_3$

$$=2\begin{vmatrix}65 & 27 & \log b\\ 65 & 27 & \log b\\ 97 & 40 & \log c\end{vmatrix}=0$$

9. (c) **10. (a)** **11. (a)**

12. (c) : $\angle AOB=60°$

$$\frac{PA}{r}=\tan 60°=\sqrt{3}\ \Rightarrow\ PA=\sqrt{3}r$$



$$\sqrt{x_1^2+y_1^2-r^2}=\sqrt{3}r\ \Rightarrow\ x_1^2+y_1^2=4r^2$$

$\Rightarrow$ Locus of $P(x_1,y_1)$ is $x^2+y^2=4r^2$

13. (d) : $x^2=\dfrac{-1\pm i\sqrt{3}}{2}=\omega,\omega^2$

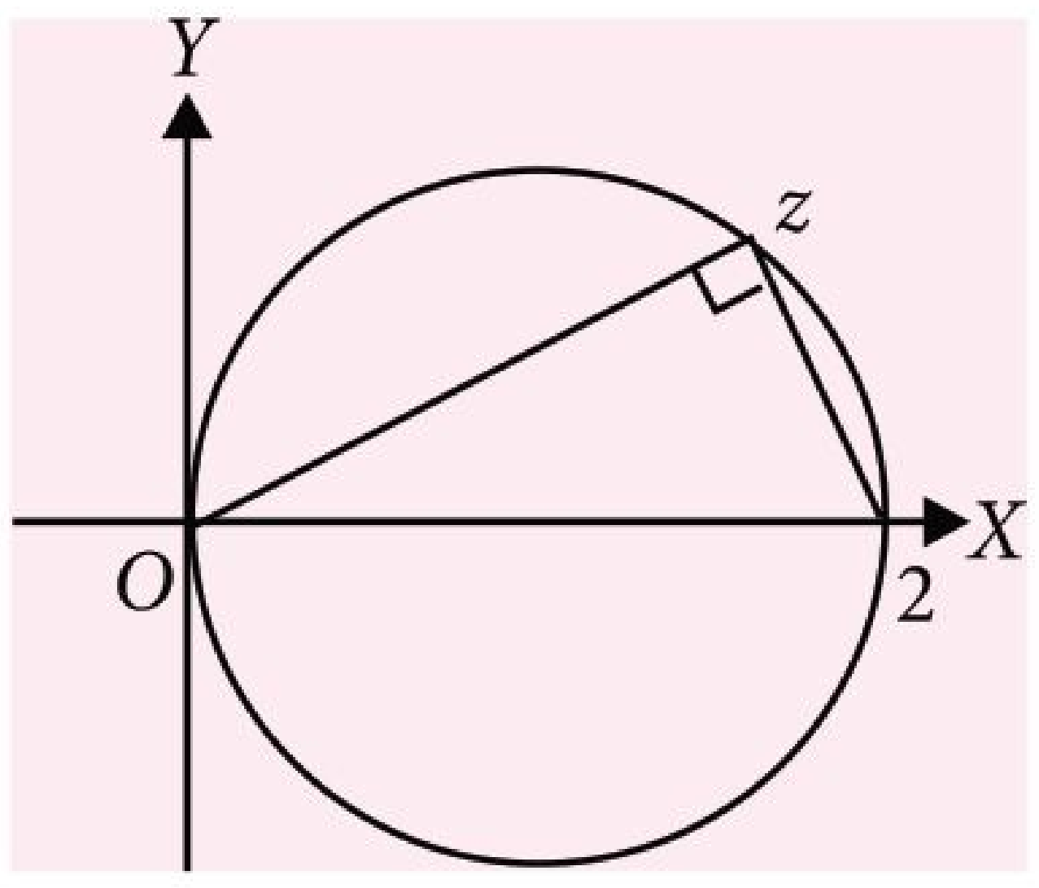
$x^6=\omega^6,(\omega^2)^6$ *i.e.* $x^6=1\ \Rightarrow\ x_1^6+x_2^6+x_3^6+x_4^6=4$

14. (d) : Note that $|z-1|=1$ represents a circle with the segment joining $z=0$ and $z=2+0i$ as a diameter.

If z lies on the circle, then

$$\arg\left(\frac{z-2}{z-0}\right)=\frac{\pi}{2}\Rightarrow\frac{z-2}{z}$$

is purely imaginary.



15. (b) : Let X be the number of coins showing heads. Then X follows a binomial distribution with parameters $n=100$ and p.

Since $P(X=50)=P(X=51)$, we get

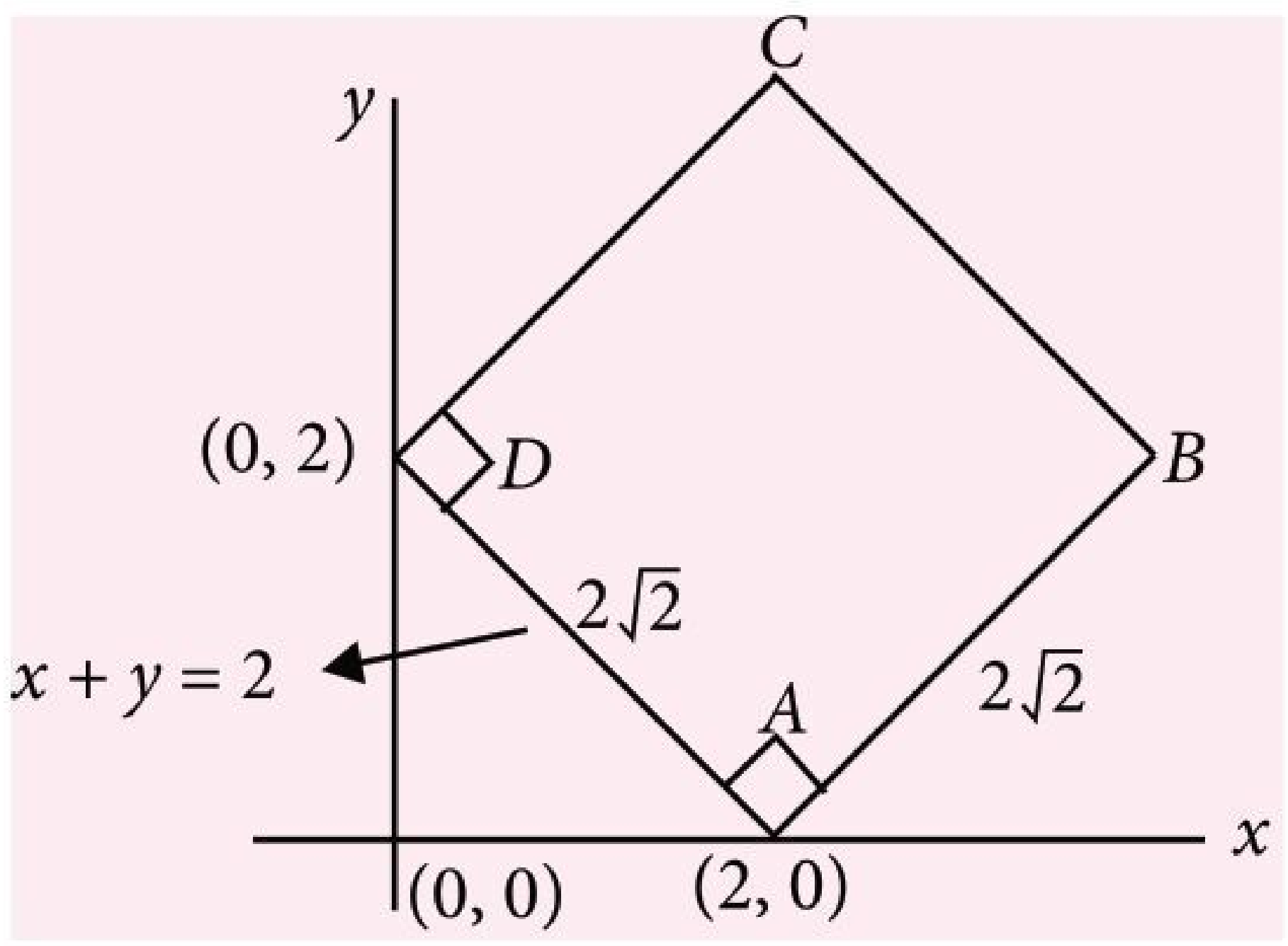
$$^{100}C_{50}p^{50}(1-p)^{50}={}^{100}C_{51}p^{51}(1-p)^{49}$$

$$\Rightarrow\ \frac{100!}{50!50!}\cdot\frac{51!49!}{100!}=\frac{p}{1-p}$$

$$\Rightarrow\ \frac{51}{50}=\frac{p}{1-p}\Rightarrow 51-51p=50p\ \Rightarrow\ p=\frac{51}{101}$$

16. (b) : Clearly, maximum value of p = perpendicular distance from (0, 0) to AD + side of the square

$$=\frac{2}{\sqrt{2}}+2\sqrt{2}=3\sqrt{2}$$



17. (b) : By definition of the hyperbola

$SP=e$(distance of P from $x=a/\sqrt{2}$

$=\sqrt{2}\,|x-(a/\sqrt{2})|$

Similarly, $S'P=\sqrt{2}\,|x+(a/\sqrt{2})|$

So that $SP\cdot SP'=2|x^2-(a^2/2)|=2x^2-a^2$

$=x^2+y^2=(CP)^2$

($\because$ P lies on the hyperbola $x^2-y^2=a^2$)

18. (c)

19. (a) : Note that $f(x)=\begin{cases}-1, & \text{if } x<0\\ 0, & \text{if } x=0\\ 1, & \text{if } x>0\end{cases}$

If $0\le a<b$, then $I=\int_a^b dx=b-a=|b|-|a|$

If $a<b<0$, then

$I=\int_a^b(-1)dx=-b+a=|b|-|a|$.

If $a<0<b$, then

$$I=\int_a^b f(x)dx=\int_a^0 f(x)dx+\int_0^b f(x)dx=|b|-|a|$$

20. (a) : We have

$$A^2=\begin{bmatrix}\omega & 0\\ 0 & \omega\end{bmatrix}\begin{bmatrix}\omega & 0\\ 0 & \omega\end{bmatrix}=\begin{bmatrix}\omega^2 & 0\\ 0 & \omega^2\end{bmatrix},$$

and $A^3 = A^2A = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$

$= \begin{bmatrix} \omega^3 & 0 \\ 0 & \omega^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \qquad [\because \omega^3 = 1]$

Now, $A^{100} = A^{99}A = (A^3)^{33}A = I^{33}A = A.$

21. (c) **22. (c)**

23. (a) : We have

$P(B \cap C') = P[(A \cup A') \cap (B \cap C')]$

$= P(A \cap B \cap C') + P(A' \cap B \cap C') = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

Now, $P(B \cap C) = P(B) - P(B \cap C') = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

24. (c) : Let $\tan^{-1}\frac{1}{3} = \alpha$ and $\tan^{-1}2\sqrt{2} = \beta$. Then

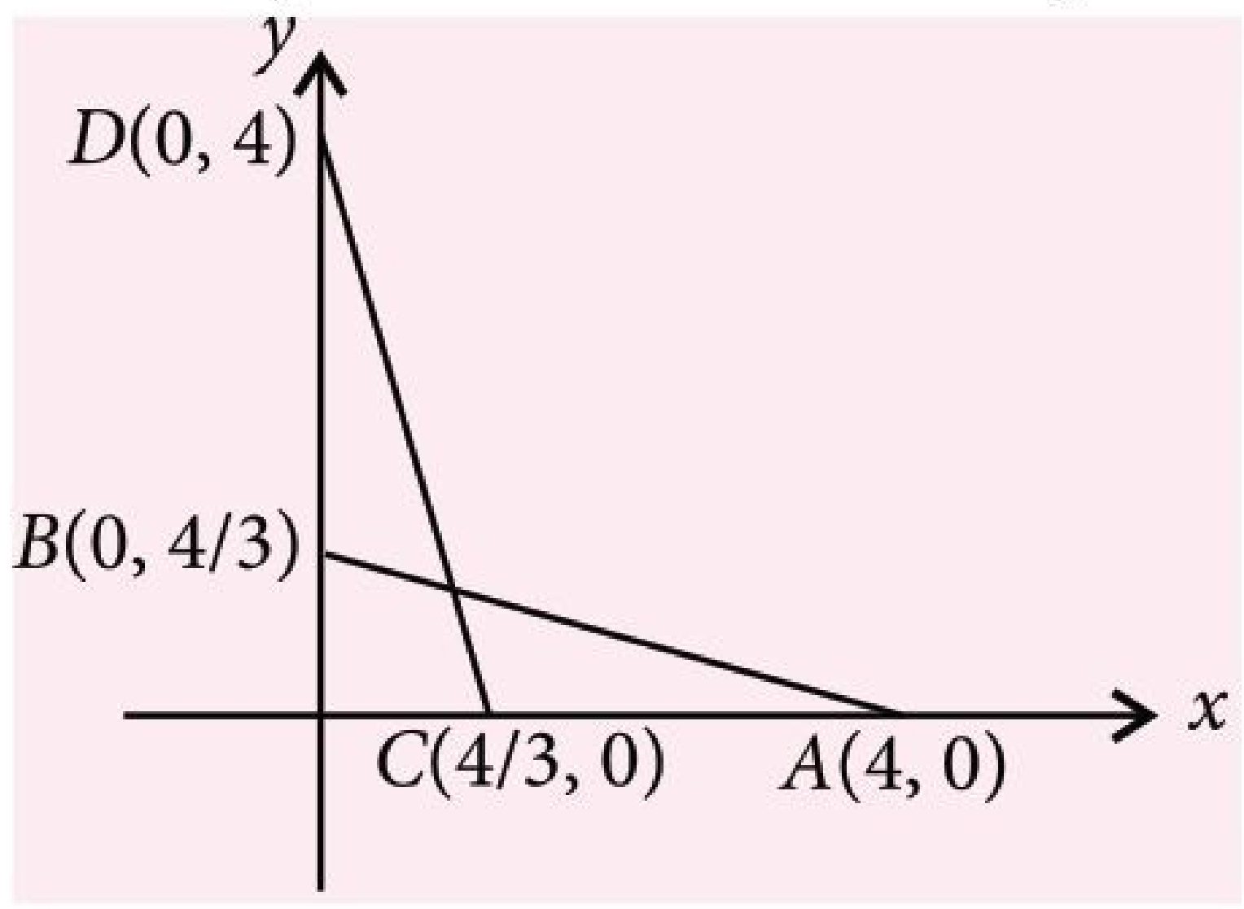
$\tan\alpha = \frac{1}{3}$ and $\tan\beta = 2\sqrt{2}$, so that

$\sin\left(2\tan^{-1}\left(\frac{1}{3}\right)\right) + \cos(\tan^{-1}2\sqrt{2}) = \sin 2\alpha + \cos\beta$

$= \frac{2\tan\alpha}{1+\tan^2\alpha} + \frac{1}{\sqrt{1+\tan^2\beta}} \quad \left[\because -\frac{\pi}{2} < \beta < \frac{\pi}{2}\right]$

$= \frac{2\left(\frac{1}{3}\right)}{1+\left(\frac{1}{9}\right)} + \frac{1}{\sqrt{1+8}} = \frac{2}{3}\cdot\frac{9}{10} + \frac{1}{3} = \frac{3}{5} + \frac{1}{3} = \frac{14}{15}$

25. (a) : $AB : x + 3y = 4$; $CD : 3x + y = 4$



Co-ordinates of centre $= \left(\frac{8}{3}, \frac{8}{3}\right)$

Radius $= \sqrt{\left(\frac{8}{3} - \frac{4}{3}\right)^2 + \left(\frac{8}{3} - 0\right)^2} = \frac{4\sqrt{5}}{3}$

26. (a)

27. (a, b, c) : Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$, we get

$\left(\frac{dy}{dx}\right)^2 = \frac{x}{a}$; $dy = \pm\frac{\sqrt{x}}{\sqrt{a}}dx \Rightarrow y = \pm\frac{1}{\sqrt{a}}\frac{2}{3}x^{3/2} - c$

$\Rightarrow (y+c)^2 = \frac{4}{9a}x^3 \Rightarrow 9a(y+c)^2 = 4x^3$

28. (a, c, d)

29. (a, b, c) : Given

$\left(\frac{a+b+c+d}{4}\right)^2 \le \frac{a^2+b^2+c^2+d^2}{4}$

$\Rightarrow \left(\frac{8-e}{4}\right)^2 \le \frac{16-e^2}{4} \Rightarrow 4 - e + \frac{e^2}{16} \le 4 - \frac{e^2}{4}$

$\Rightarrow \frac{e}{16}(5e - 16) \le 0. \quad \therefore\ 0 < e \le \frac{16}{5}$

30. (b, d) : $\sin\theta + \sqrt{3}\cos\theta = -2 - (x^2 - 6x + 9)$
$= -2 - (x - 3)^2$

$\because\ \sin\theta + \sqrt{3}\cos\theta \le -2$

But minimum value of $\sin\theta + \sqrt{3}\cos\theta = -2$ and then $x = 3$

$\therefore\ x = 3$ and $\cos\left(\theta - \frac{\pi}{6}\right) = -1$

$\theta - \frac{\pi}{6} = \pi, 3\pi;\ \theta = \frac{7\pi}{6}, \frac{19\pi}{6}$

31. (c) : Given $(\sqrt{2} + \sqrt[5]{3})^{120}$

$\therefore\ T_{r+1} = {}^{120}C_r\left(2^{\frac{1}{2}}\right)^{120-r}\cdot\left(3^{\frac{1}{5}}\right)^r = {}^{120}C_r 2^{60-\frac{r}{2}}\cdot 3^{\frac{r}{5}}$

For rational terms $\frac{r}{2}, \frac{r}{5}$ are integers

$\therefore$ r = the multiple of the L.C.M of (2, 5) = 10

As, $0 \le r \le 120$

$\therefore$ $r = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 120$

$\therefore$ Number of rational terms = 13

$\therefore$ Number of irrational terms = 121 – 13 = 108

32. (b) : The given expansion $(2^{1/5} + 7^{1/7})^{105}$

$\therefore T_{r+1} = {}^{105}C_r(2^{1/5})^{105-r}\cdot(7^{1/7})^r$

$= {}^{105}C_r 2^{21-\frac{r}{5}}\cdot 7^{\frac{r}{7}}$

For rational terms r is the multiple of $7 \times 5 = 35$

$\therefore\ 0 \le r \le 105\ \therefore\ r = 0, 35, 70, 105$

So, sum of rational terms is $T_1 + T_{36} + T_{71} + T_{106}$

$= {}^{105}C_0\, 2^{21}\cdot 7^0 + {}^{105}C_{35}\, 2^{14}\, 7^5 + {}^{105}C_{70}\, 2^7\cdot 7^{10}$
$+ {}^{105}C_{105}\cdot 2^0\cdot 7^{15}$

33. (a) : Given $(7^{1/3} + 11^{1/9})^{5832}$

$T_{r+1} = {}^{5832}C_r 7^{\frac{5832-r}{3}}\cdot 11^{\frac{r}{9}}$

Now T_{r+1} will be rational if $\frac{r}{9}, \frac{r}{3}$ are integers

∴ r is a multiple of 9

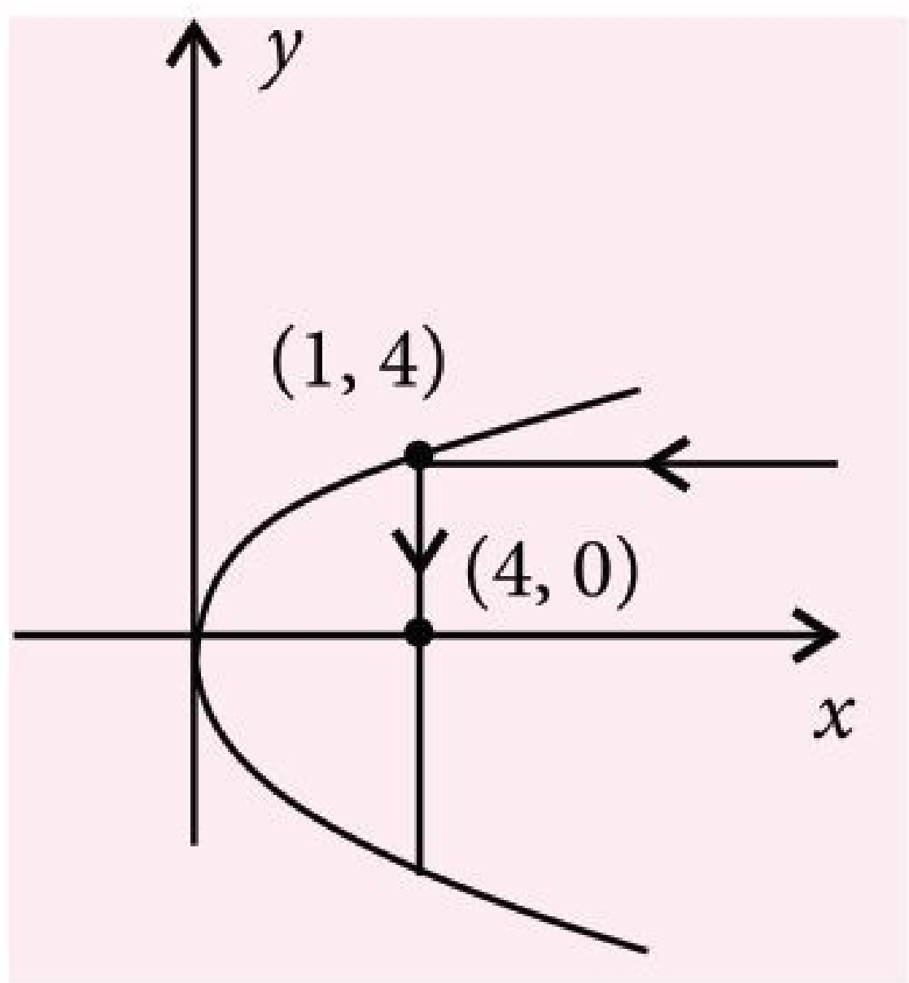
∴ $0 \le r \le 5832$ and $0 \le \frac{r}{9} \le 648$

∴ $\frac{r}{9} = 0, 1, 2, 3, \ldots, 648$

∴ The total number of terms free from radicals
$= 648 + 1 = 649.$

34. (b) : Given $y^2 = 16x$
Focus (4, 0) and the point of intersection of $y = 4$ and the parabola $y^2 = 16x$ is (1, 4)

∴ Equation of the reflected ray is given by



$$y - 0 = \frac{4-0}{1-4}(x-4) \Rightarrow 3y + 4x = 16$$

35. (a) : The given equation of the parabola is

$$y^2 - 10y - 8x + 41 = 0 \Rightarrow (y-5)^2 = 8(x-2)$$

$\Rightarrow$ $Y^2 = 8X$

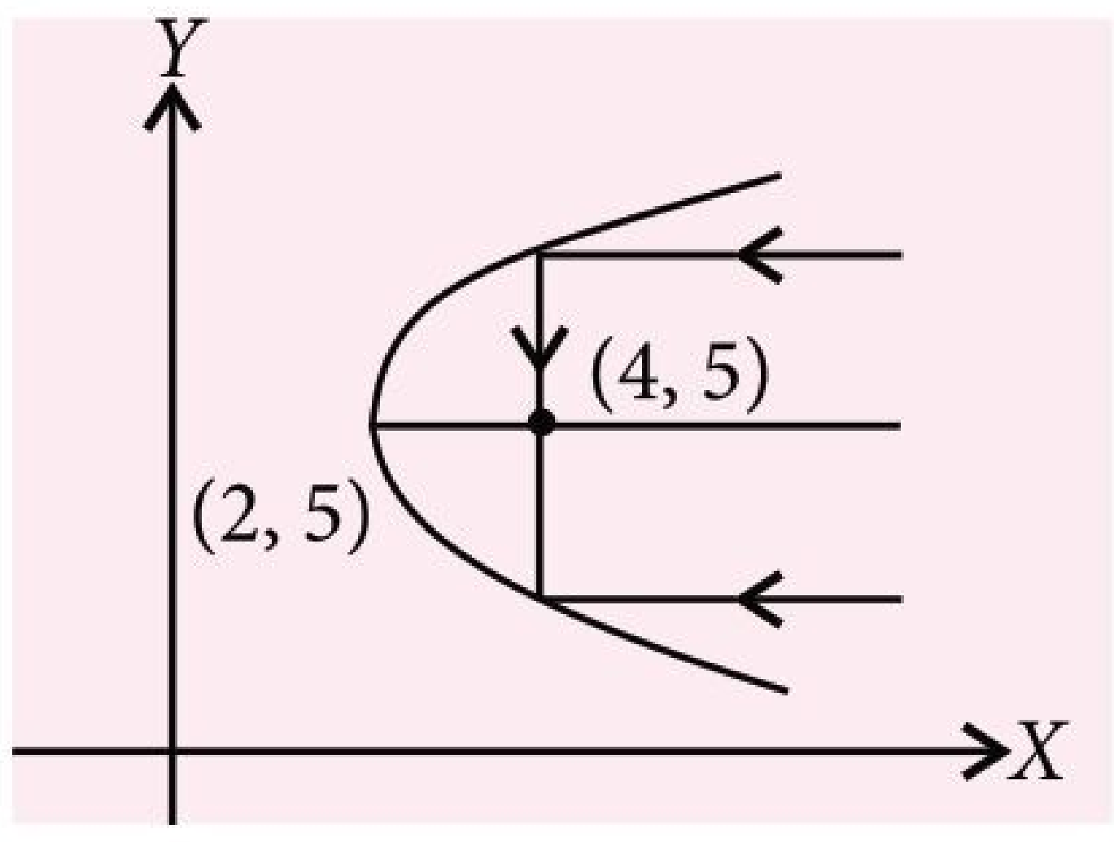
where $Y = y - 5$ and $X = x - 2$

$\Rightarrow$ Focus $(a, 0) = (2, 0)$

$\Rightarrow$ $X = 2 \Rightarrow x - 2 = 2 \Rightarrow x = 4$

and $Y = 0 \Rightarrow y - 5 = 0 \Rightarrow y = 5$

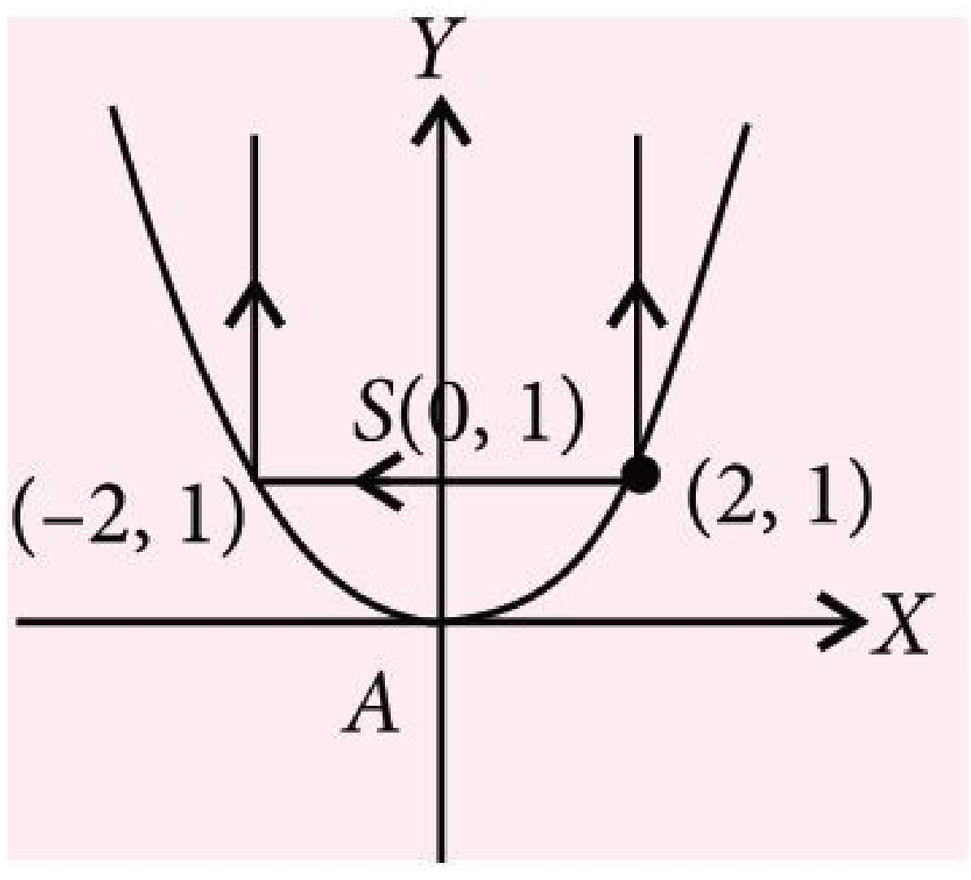
∴ After reflection, the ray must pass through the focus (4, 5)



36. (c) : The given equation of parabola is $x^2 = 4y$

∴ Focus (0, 1) and the point of intersection of $x = 2$ and $x^2 = 4y$ is (2, 1)

Now equation of the first reflection is $y = 1$ which is perpendicular to the incident ray, hence the second reflected ray is parallel to incident ray and pass through the point (–2, 1) is $x = -2$ *i.e.*, $x + 2 = 0$.



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JEE WORK OUTS

One or More Than One Option(s) Correct Type

1. Given the following five curves (C_1, C_2, C_3, C_4 and C_5)

C_1: Parabola with vertex at origin and focus on negative x-axis

C_2: Parabola with vertex at origin and focus on positive x-axis

C_3: Ellipse with centre at origin and focii on x-axis.

C_4: Ellipse with centre at origin and focii on y-axis.

C_5: Hyperbola with centre at origin and focii on y-axis.

Let 'a' and 'b' be non-zero real numbers. Then the diagram representing the curves $ax + by^2 = 0$ and $ax^2 + by^2 = 1$ in the same coordinate plane can be

(a) C_2 and C_5 (b) C_1 and C_5
(c) C_1 and C_3 (d) C_1 and C_4

2. The eighth term of the sequence 1440, 1716, 1848,..., whose terms are formed by multiplying corresponding terms of two A.P's is

(a) 348 (b) 2048 (c) 2134 (d) 512

3. The minimum number of points required to draw a parabola uniquely is

(a) 2 (b) 3 (c) 4 (d) 5

4. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c}$ is also a unit vector and α, β, γ are angles between the vectors $\vec{a}, \vec{b}; \vec{b}, \vec{c}$ and $\vec{c}, \vec{a}$ respectively then among α, β, γ

(a) all are acute angles
(b) atleast one is right angle
(c) atleast one is more than right angle
(d) none of these

5. Let A and B be two $n \times n$ real matrices such that $AB + A + B = 0$ then

(a) $AB = BA$ (b) $AB + BA = 0$
(c) $(A + I)(B + I) = 2I$ (d) $(A + I) = (B + I)$

6. Let X and Y are two 2×2 matrices with integer entries such that $\begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} = X^3 + Y^3$, then

(a) $X = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$ (b) $X + Y = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix}$
(c) $Y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $X - Y = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$

7. If $a + b = 1$ and $a^2 + b^2 = 2$, then

(a) $a^3 + b^3 = 5/2$ (b) $a^4 + b^4 = 7/2$
(c) $a^5 + b^5 = 19/4$ (d) $a^7 + b^7 = 71/8$

8. The vertices of a triangle which has the three lines $y = 2x - 2$, $x + y = 4$ and $y = 2$ as the medians and line $4y = x + 3$ as one of the altitudes are

(a) (1, 0) (b) (0, 4) (c) (5, 2) (d) (1, 2)

9. If $S_n = \sum_{k=0}^{n} \frac{{}^{n+k}C_k}{2^k}$ then

(a) $S_{20} = 2S_{19}$ (b) $S_{14} = \frac{1}{2} \cdot S_{13}$
(c) $S_9 = 512$ (d) $S_{10} = 2048$

10. Which of the following statements are true for $f(x) = [\sin[x]]$ (where $[\cdot]$ is greatest integer function)?

(a) $f(1+) = f(2+) = f(3+)$
(b) $f(4+) < f(3+)$
(c) $f(2-) = f(3-)$ (d) $f(0-) = f(5-)$

11. If a, b, c and d be lengths of tangent, sub-tangent, normal & sub-normal respectively for the curve $y = x + \frac{x^2}{2} + \frac{x^3}{3}$ then

(a) $ac^2 = bd^2$ (b) $ab^2 = cd^2$

(c) $bc^2 = a^2d$ (d) None of these

12. If sum of the coefficients of x^t & x^{t+1} in $\sum_{r=0}^{n}(1+x)^r$ be ${}^{n+2}C_{2013}$ then

(a) t is odd (b) t is prime

(c) t is even

(d) t is composite number $(n \geq 2012)$

13. If $z = x + iy$ and $\operatorname{cosec}^{-1}\left(\frac{z}{1+i}\right)$ be defined then

(a) $z \in$ 1st or 3rd Quadrant

(b) $z \in$ 2nd or 4th Quadrant

(c) z lies on real axis

(d) z lies on imaginary axis

14. Consider the circle $x^2 + y^2 - 8x - 18y + 93 = 0$ with centre 'C' and point $P(2, 5)$ outside it. From the point P, a pair of tangents PQ and PR are drawn to the circle with S as the midpoint of QR. The line joining P to C intersects the given circle at A and B. Which of the following hold(s) good ?

(a) CP is the arithmetic mean of AP and BP

(b) PR is the geometric mean of PS and PC

(c) PS is the harmonic mean of PA and PB

(d) The angle between the two tangents from P is $\tan^{-1}\left(\frac{3}{4}\right)$

15. If $x\cos\alpha + y\sin\alpha = x\cos\beta + y\sin\beta = 2a$ and $2\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)=1$ then

(a) $y^2 = 4a(a - x)$

(b) $\cos\alpha + \cos\beta = \cos\alpha\cos\beta$

(c) $\cos\alpha\cdot\cos\beta = \frac{4a^2+y^2}{x^2+y^2}$

(d) $\cos\alpha + \cos\beta = \frac{4ax}{x^2+y^2}$

16. An ellipse whose major axis is parallel to x-axis is such that the segments of a focal chord are 1 and 3 units. The lines $ax + by + c = 0$ are the chords of the ellipse such that a, b, c are in A.P· and bisected by the point at which they are concurrent. The equation of auxiliary circle is $x^2 + y^2 + 2\alpha x + 2\beta y - 2\alpha - 1 = 0$. Then

(a) The locus of perpendicular tangents to the ellipse is $x^2 + y^2 = 7$

(b) Length of the double ordinate which is conjugate to directrix is 3

(c) Area of the auxiliary circle is 2π

(d) Eccentricity of the ellipse is $\frac{1}{2}$

17. The projection of line $3x - y + 2z - 1 = 0 = x + 2y - z - 2$ on the plane $3x + 2y + z = 0$ is

(a) $\frac{x+1}{11}=\frac{y-1}{-9}=\frac{z-1}{-15}$

(b) $3x - 8y + 7z + 4 = 0 = 3x + 2y + z$

(c) $\frac{x+12}{11}=\frac{y+8}{-9}=\frac{z+14}{15}$

(d) $\frac{x+12}{11}=\frac{y+8}{-9}=\frac{z+14}{-15}$

18. If the median through A of a ΔABC having vertices $A \equiv (2, 3, 5)$, $B \equiv (-1, 3, 2)$ and $C \equiv (\lambda, 5, \mu)$ is equally inclined to the axes, then

(a) $\lambda = 7$ (b) $\mu = 10$ (c) $\lambda = 10$ (d) $\mu = 7$

19. The integral of $\frac{1}{\sin^2 x + \tan^2 x}$ must be

(a) $-\frac{1}{2}\left[\tan x + \frac{1}{\sqrt{2}}\tan^{-1}\left(\tan\frac{x}{\sqrt{2}}\right)\right]+c$

(b) $-\frac{1}{2}\left[\cot x + \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\tan x\right)\right]+c$

(c) $-\left[\cot x - \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\tan x\right)\right]+c$

(d) $-\frac{1}{2}\left[\cot x - \frac{1}{\sqrt{2}}\cot^{-1}\left(\frac{1}{\sqrt{2}}\tan x\right)\right]+c$

20. The line $y = x + 5$ touches

(a) the parabola $y^2 = 20x$

(b) the ellipse $9x^2 + 16y^2 = 144$

(c) the hyperbola $\frac{x^2}{29}-\frac{y^2}{4}=1$

(d) the circle $x^2 + y^2 = 25$

Comprehension Type

Paragraph for Q. No. 21 to 23

The normal at any point (x_1, y_1) of curve is a line perpendicular to tangent at the point (x_1, y_1). In case of parabola $y^2 = 4ax$, the equation of normal is $y = mx - 2am - am^3$ (m is slope of normal). In case of rectangular hyperbola $xy = c^2$, the equation of normal at $(ct, c/t)$ is $xt^3 - yt - ct^4 + c = 0$. The shortest distance between any two curves always exist along the common normal.

21. If normal at (5, 3) of rectangular hyperbola $xy - y - 2x - 2 = 0$ intersect it again at a point

(a) $(-1, 0)$ (b) $(-1, 1)$
(c) $(0, -2)$ (d) $(3/4, -14)$

22. The shortest distance between the parabolas $2y^2 = 2x - 1, 2x^2 = 2y - 1$ is

(a) $2\sqrt{2}$ (b) $\frac{1}{2\sqrt{2}}$
(c) 4 (d) $\sqrt{\frac{36}{5}}$

23. Number of normals drawn from $\left(\frac{7}{6}, 4\right)$ to parabola $y^2 = 2x - 1$ is

(a) 1 (b) 2 (c) 3 (d) 4

Paragraph for Q. No. 24 and 25

A right circular cone with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant $= k > 0$). Suppose that $r(t)$ is the radius of liquid cone at time t.

24. The time after which the cone is empty is

(a) $H/2k$ (b) H/k
(c) $H/3k$ (d) $2H/k$

25. The radius of water cone at $t = 1$ is

(a) $R[1 - k/H]$ (b) $R[1 - H/k]$
(c) $R[1 + H/k]$ (d) $R[1 + k/H]$

Paragraph for Q. No. 26 to 28

For a finite set A, let $|A|$ denote the number of elements in the set A. Also let F denote the set of all functions $f: \{1, 2, 3, ..., n\} \to \{1, 2, ..., k\}$ $(n \geq 3, k \geq 2)$ satisfying $f(i) \neq f(i + 1)$ for every i, $1 \leq i \leq n - 1$.

26. $|F| =$

(a) $k^n(k - 1)$ (b) $k(k - 1)^{n-2}$
(c) $k^{n-1}(k - 1)$ (d) None of these

27. If $c(n, k)$ denote the number of functions in F satisfying $f(n) \neq f(1)$, then for $n \geq 4$, $c(n, k) =$

(a) $k(k - 1)^n - c(n - 1, k)$
(b) $k(k - 1)^n - c(n - 1, k - 1)$
(c) $k^{n-1}(k - 1) - c(n - 1, k)$
(d) None of these

28. For $n \geq k$, $c(n, k)$, where $c(n, k)$ has the same meaning as in given, equals

(a) $k^n + (-1)^n (k - 1)$
(b) $(k - 1)^n + (-1)^{n-1}(k - 1)$
(c) $(k - 1)^n + (-1)^n (k + 1)$
(d) None of these

Matrix Match Type

29. Match the following.

Column I		Column II	
(A)	The maximum value of $\sin(\cos x) + \cos(\sin x)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is	(p)	$\cos(\cos 1)$
(B)	The minimum value of $\sin(\cos x) + \cos(\sin x)$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is	(q)	$1 + \cos 1$
(C)	The maximum value of $\cos(\cos(\sin x))$ is	(r)	$\cos 1$
(D)	The minimum value of $\cos(\cos(\sin x))$ is	(s)	$1 + \sin 1$

30. In the column-I and column-II differential equations and their corresponding solutions are given. Match them.

Column I		Column II	
(A)	$(\cos x)\frac{dy}{dx} + y\sin x = 1$	(p)	$y = \frac{\sin x + c}{1 + x^2}$
(B)	$x\frac{dy}{dx} + y = x\sin x$	(q)	$y = \sin x + c\cos x$
(C)	$(1 + x^2)\frac{dy}{dx} + 2xy = \cos x$	(r)	$xy = \sin x - x\cos x + c$
(D)	$\frac{dy}{dx} + (2\tan x)y = \sin x$	(s)	$y = \cos x + c\cos^2 x$

31. Match the following.

Column I		Column II	
(A)	The minimum value of ab if roots of the equation $x^3 - ax^2 + bx - 2 = 0$ are positive, is	(p)	24
(B)	The number of divisors of the form $12\lambda + 6 (\lambda \in N)$ of the number 25200 are	(q)	3
(C)	The number of quadrilateral formed in an octagon having two adjacent sides common with the polygon are	(r)	12
(D)	The number of solution of the equation $[\cos x] = \tan x$, where $[\cdot]$ denotes greatest integer function $\forall\, x \in [0, 6\pi]$ are	(s)	18

Numerical Answer Type

32. If length of the side BC of a ΔABC is 4 cm and $\angle BAC = 120°$, then the distance between incentre and excentre of the circle touching the side BC internally is

33. In parallelogram $ABCD$, P is any point inside parallelogram such that sum of area of ΔAPD and ΔBPC is 3 sq. units, then area of parallelogram $ABCD$ is

34. $A = (-3, 0)$ and $B = (3, 0)$ are the extremities of the base AB of triangle PAB. If the vertex P varies such that the internal bisector of angle APB of the triangle divides the opposite side AB into two segments AD and BD such that $AD : BD = 2 : 1$, then the maximum value of the length of the altitude of the triangle drawn through the vertex P is

35. Find the maximum value of $(\log_{2^{1/5}} a)\cdot(\log_{2^{1/2}} b)$. It is given that coefficient of 2nd, 3rd and 4th term in expansion of $(a + b)^n$ are in A.P and the value of 3rd term is equal to 84 $(a, b > 1)$.

36. Let $S = \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\ldots..$

$$+\sqrt{1+\frac{1}{1999^2}+\frac{1}{2000^2}},$$

then find $|2000(S - 2000)|$.

37. Consider two polynomials $f(x)$ and $g(x)$ as $g(x) = \sum_{r=0}^{200} \alpha_r x^r$ and $f(x) = \sum_{r=0}^{200} \beta_r x^r$. Given:

(i) $\beta_r = 1 \ \forall\ r \geq 100$ (ii) $f(x + 1) = g(x)$.

Let $A = \sum_{r=100}^{200} \alpha_r$. Find the remainder when A is divided by 15.

38. A sequence is obtained by deleting all perfect squares from set of natural numbers. The remainder when the 2003rd term of new sequence is divided by 2048, is

39. If $\alpha = e^{i2\pi/7}$ and $f(x) = A_0 + \sum_{k=1}^{20} A_k x^k$ and the value of $f(x) + f(\alpha x) + f(\alpha^2 x) + \ldots + f(\alpha^6 x)$ is $k(A_0 + A_7 x^7 + A_{14} x^{14})$, then find the value of k.

40. Let $X = \{1, 2, 3, \ldots 100\}$ and Y be a subset of X such that the sum of no two elements in Y is divisible by 7. If the maximum possible number of element in Y is $40 + \lambda$ then λ is

SOLUTIONS

1. (a,c,d) : 'a' and 'b' both cannot be negative. Either one is positive and other is negative or both are positive.

2. (a) : n^{th} term of such a sequence is of the form $an^2 + bn + c$.

3. (c) : There are 4 independent parameters in the general 2nd degree conic equation for parabola $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ and $h^2 = ab$.

4. (c) : $\cos\alpha + \cos\beta + \cos\gamma = -1$

5. (a) : $A + I$ and $B + I$ are inverses of each other.

6. (a, b) : If A is the given matrix then $A^3 + 3A^2 + 2A = 0$, *i.e.*, $A = (A + I)^3 - I$. Hence $X = (A + I)^3$ and $Y = -I$

7. (a, b, c, d) : Let $S_n = a^n + b^n$ then use the recursive relation $S_{n+2} - S_{n+1} - \frac{1}{2} S_n = 0$.

8. (a, b, c)

9. (a, c) : Note that ${}^{n+k}C_k = {}^{n+k}C_n$, $S_n = 2^n$.

10. (a,b,c,d) :

$f(0-) = [\sin(-1)] = [-\sin 1] = -1$

$[\because 1 \text{ radian} = 57°(\text{approx.})]$

$f(2-) = [\sin 1] = 0\,; f(3-) = [\sin 2] = 0$

$f(5-) = [\sin 4] = -1\,; f(1+) = [\sin 1] = 0$

$f(2+) = [\sin 2] = 0\,; f(3+) = [\sin 3] = 0$

$f(4+) = [\sin 4] = -1$

11. (c) : Length of tangent $= y\sqrt{1+\left(\frac{dx}{dy}\right)^2}$

Length of normal $= y\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

Length of sub-tangent $= \left|y\cdot\frac{dx}{dy}\right|$

& Length of sub-normal $= \left|y\cdot\frac{dy}{dx}\right|$ for any curve

Also $\frac{(\text{Length of normal})^2}{(\text{Length of tangent})^2} = \left(\frac{dy}{dx}\right)^2$

$= \frac{\text{Length of sub-normal}}{\text{Length of sub-tangent}} \Rightarrow \quad \frac{c^2}{a^2} = \frac{d}{b} \Rightarrow bc^2 = a^2 d$

12. (a, b) :

$\because \sum_{r=0}^{n} (1+x)^r = 1 + (1+x) + (1+x)^2 + \ldots.. + (1+x)^n$

$= \frac{1\cdot\{(1+x)^{n+1}-1\}}{(1+x)-1} = \frac{(1+x)^{n+1}-1}{x}$

$\therefore$ Coefficient of x^t = co-efficient of x^{t+1} in $\{(1+x)^{n+1}-1\} = {}^{n+1}C_{t+1}$

Similarly, co-efficient of $x^{t+1} = {}^{n+1}C_{t+2}$

According to problem, ${}^{n+1}C_{t+1} + {}^{n+1}C_{t+2} = {}^{n+2}C_{2013}$

$\Rightarrow \quad {}^{n+2}C_{t+2} = {}^{n+2}C_{2013}$

$\Rightarrow \quad t = 2011$

13. (a) : $\operatorname{cosec}^{-1}\dfrac{z}{1+i} = \operatorname{cosec}^{-1}\left\{\dfrac{(x+iy)(1-i)}{1-i^2}\right\}$

$= \operatorname{cosec}^{-1}\left\{\dfrac{(x+y)}{2} + i\left(\dfrac{y-x}{2}\right)\right\}$, which will be defined.

if $\dfrac{y-x}{2} = 0$ & $\dfrac{x+y}{2} \le -1$ or $\dfrac{x+y}{2} \ge 1$

i.e., $x = y$ & $\{x + y \le -2$ or $x + y \ge 2\}$

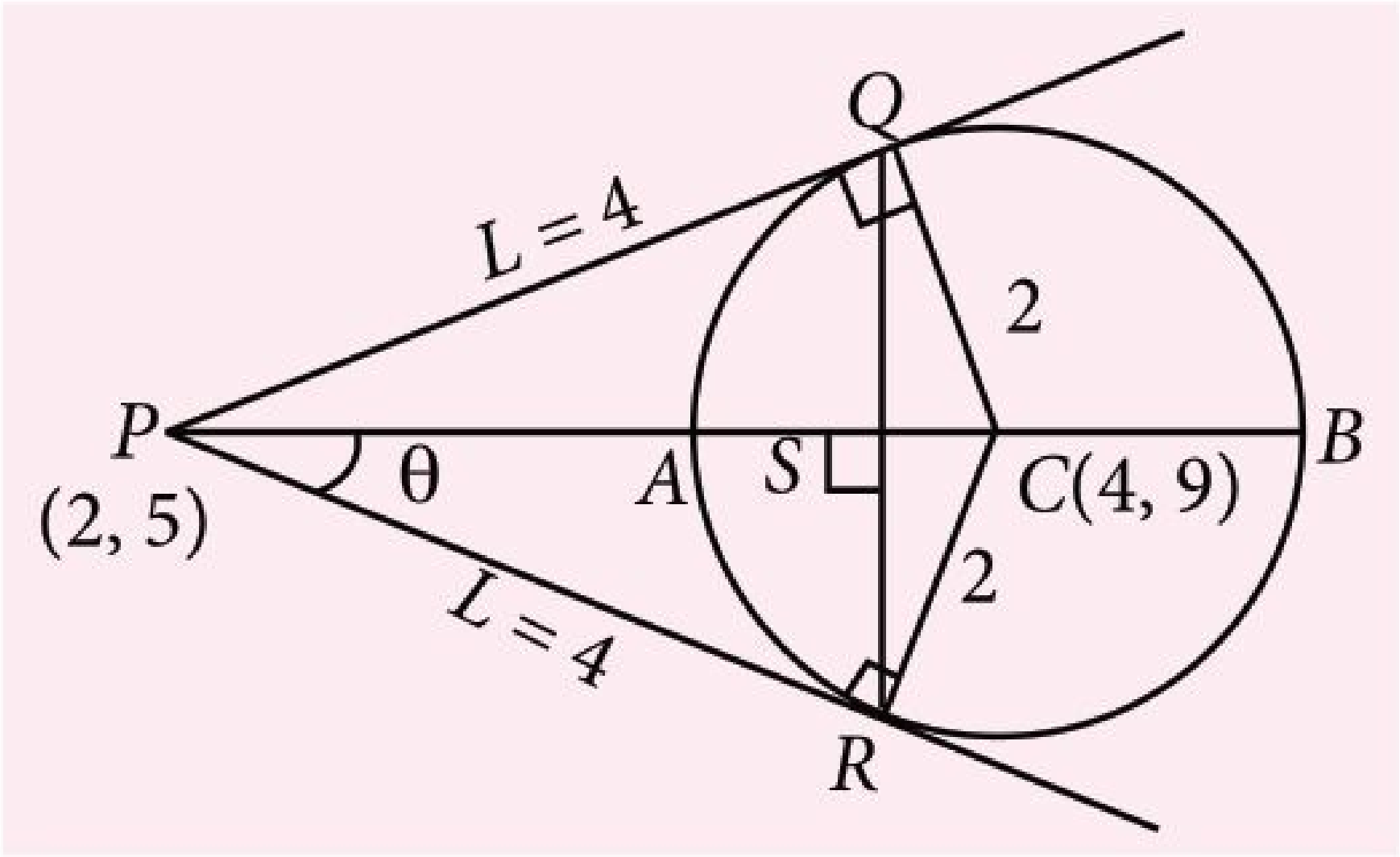
$\Rightarrow \quad (x \le -1, y \le -1)$ or $(x \ge 1, y \ge 1)$

$z \in$ 1st or 3rd quadrant.

14. (a, b, c) : We have, radius $= \sqrt{16+81-93} = 2$

$$CP = \sqrt{20},\ AP = \sqrt{20}-2;\ BP = \sqrt{20}+2$$

$\Rightarrow \quad CP = \dfrac{AP+BP}{2} \Rightarrow$ (a) is correct



Now, let $L = PR = \sqrt{(PC)^2 - r^2} = \sqrt{20-4} = 4 = PQ$

and $\tan\theta = \dfrac{2}{4} = \dfrac{1}{2}$

Also, $\cos\theta = \dfrac{PS}{PR}$

$\Rightarrow \quad PS = PR\cos\theta = 4\cdot\left(\dfrac{2}{\sqrt{5}}\right) = \dfrac{8}{\sqrt{5}}$

Harmonic mean between *PA* and *PB*

$$= \frac{2(\sqrt{20}-2)(\sqrt{20}+2)}{2\sqrt{20}} = \frac{16}{2\sqrt{5}} = \frac{8}{\sqrt{5}} = PS$$

$\Rightarrow \quad$ (c) is correct

Hence $(PS)(PC) = \left(\dfrac{8}{\sqrt{5}}\right)(\sqrt{20}) = 16 = (PR)^2$

$\Rightarrow \quad$ *PR* is the Geometric Mean of *PS* and *PC*

$\Rightarrow \quad$ (b) is correct

Now angle between the two tangents is 2θ, then

$$2\tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}\left(\frac{2\cdot\frac{1}{2}}{1-\frac{1}{4}}\right) \quad \left(\text{As } \tan\theta = \frac{1}{2}\right)$$

$$= \tan^{-1}\left(\frac{4}{3}\right).$$

$\Rightarrow \quad$ (d) is incorrect.

15. (a, b, d) : α and β satisfy $x\cos\theta + y\sin\theta = 2a$

$\Rightarrow \quad (x^2 + y^2)\cos^2\theta - 4ax\cos\theta + (4a^2 - y^2) = 0$

$$\cos\alpha + \cos\beta = \frac{4ax}{x^2+y^2},\ \cos\alpha\cdot\cos\beta = \frac{4a^2-y^2}{x^2+y^2}$$

$$2\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right) = 1 \Rightarrow 4\sin^2\left(\frac{\alpha}{2}\right)\sin^2\left(\frac{\beta}{2}\right) = 1$$

$\Rightarrow \quad \cos\alpha + \cos\beta = \cos\alpha\cdot\cos\beta$

$\Rightarrow \quad \dfrac{4ax}{x^2+y^2} = \dfrac{4a^2-y^2}{x^2+y^2} \Rightarrow y^2 = 4a(a-x)$

16. (b, d)

17. (a, b) : Equation of a plane passing through the line $3x - y + 2z - 1 = 0 = x + 2y - z - 2$ is

$$3x - y + 2z - 1 + \lambda(x + 2y - z - 2) = 0$$

Since it is perpendicular to the given plane

$\therefore \ \lambda = -\dfrac{3}{2}$

$\therefore$ Equation of the line of projection is

$$3x - 8y + 7z + 4 = 0 = 3x + 2y + z$$

Its direction ratios are < 11, –9, –15 > and the point (–1,1,1) lies on the line

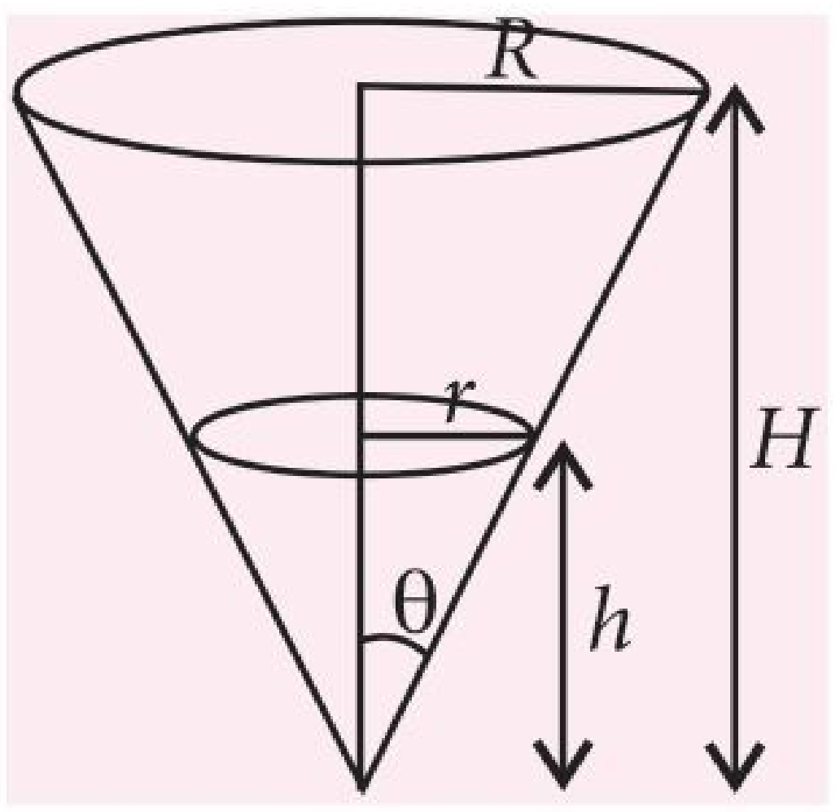
$\therefore \ \dfrac{x+1}{11} = \dfrac{y-1}{-9} = \dfrac{z-1}{-15}$ is also the equation of the line of projection.

18. (a,b) **19. (d)** **20. (a, b, c)**

21. (d) **22. (b)** **23. (a)**

(24-25)

24. (b) **25. (a)**



Let θ be the semi vertical angle of the cone so that tan θ = *R*/*H*. Let the radius and height of water cone at time *t* be *r* and *h* respectively.

So, $\tan\theta = \dfrac{r}{h}$

If *V* is the volume of water and *S* is the surface area of the cone in contact with air at time *t*, then

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3\cot\theta \quad \text{and } S = \pi r^2$$

We are given that $\dfrac{dV}{dt} \propto S$

$$\Rightarrow \frac{dV}{dt} = -kS \quad (\because V \text{ is decreasing})$$

$$\Rightarrow \frac{1}{3}\pi(3r^2)\cot\theta\frac{dr}{dt} = -k(\pi r^2) \quad \Rightarrow \frac{dr}{dt} = -k\tan\theta$$

Integrating, we get $r = -(k\tan\theta)t + C$

When $t = 0$, $r = R$, $\therefore C = R$

Thus, $r = (-k\tan\theta)t + R$

When cone is empty, $r = 0$. If T is the time taken for the cone to be empty, then $0 = (-k\tan\theta)T + R$

$$\Rightarrow T = \frac{R}{k\tan\theta} = \frac{R}{k(R/H)} = \frac{H}{k}$$

Hence, cone will be empty in time H/k.

$$r(1) = -k\tan\theta + R = -k\frac{R}{H} + R = R\left(1 - \frac{k}{H}\right)$$

26.(d) **27. (d)** **28. (d)**

29. (A) → (s); (B) → (r); (C) → (p); (D) → (r)

Let $f(x) = \sin(\cos x) + \cos(\sin x)$

f is an even function. We can take $x \in \left[0, \frac{\pi}{2}\right]$.

In $\left[0, \frac{\pi}{2}\right]$, $\sin x$ is increasing and $\cos x$ is decreasing.

Hence f is a decreasing function. Therefore, maximum value of f is $f(0) = \sin 1 + 1$ and minimum value is $f(\pi/2) = 0 + \cos 1$.

Let $g(x) = \cos(\cos(\sin x))$. Obviously g is an even periodic function of period π. Hence g takes all of its values for $x \in \left[0, \frac{\pi}{2}\right]$.

It can be seen that g is an increasing function in $\left[0, \frac{\pi}{2}\right]$.

So maximum value of g is $g(\pi/2) = \cos(\cos 1)$, and minimum value of g is $g(0) = \cos 1$.

30. (A) → (q); (B) → (r); (C) → (p); (D) → (s)

(A) $\frac{dy}{dx} + (\tan x)y = \sec x$

$$\text{I.F.} = e^{\int \tan x dx} = e^{\log \sec x} = \sec x$$

$$y \sec x = \int \sec^2 x dx = \tan x + c$$

$y = \sin x + c \cos x$

(B) $\frac{dy}{dx} + \frac{y}{x} = \sin x$, I.F. $= x$

$$yx = \int x \sin x dx + c = \sin x - x\cos x + c$$

(C) $\frac{dy}{dx} + \left(\frac{2x}{1+x^2}\right)y = \frac{\cos x}{1+x^2}$

I.F. $= 1 + x^2$

$$y(1+x^2) = \int \cos x \, dx = \sin x + c$$

(D) $\frac{dy}{dx} + (2\tan x)y = \sin x$

I.F. $= \sec^2 x$

$$y\sec^2 x = \int \sin x \sec^2 x \, dx = \sec x + c$$

$y = \cos x + c\cos^2 x$

31. (A) → (s); (B) → (r); (C) → (p); (D) → (q)

(A) $x^3 - ax^2 + bx - 2 = 0$

Let x_1, x_2, x_3 be roots

A.M. ≥ G.M.

$$\Rightarrow \left(\frac{x_1 + x_2 + x_3}{3}\right)\left(\frac{\Sigma x_1 x_2}{3}\right) \geq (x_1x_2x_3)^{1/3}(x_1x_2x_3)^{2/3}$$

$$\Rightarrow \frac{ab}{9} \geq 2 \quad \Rightarrow (ab)_{\min} = 18$$

(B) $25200 = 2^4 3^2 5^2 7^1$

Now, if divisor is of the form $12\lambda + 6$ *i.e.* $6(2\lambda + 1)$ then there must be exactly one 2 and at least one 3.

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$\therefore$ Number of divisors are $1\cdot 2\cdot(2+1)(1+1) = 12$

(C) Number of quadrilateral are $n(n-5) = 8 \times 3 = 24$

(D) $[\cos x] = \tan x$

$\tan x = -1$ and $\cos x < 0$

$\therefore$ Three solutions.

32. (8): I_1 and I are excentre and incentre of ΔABC

$II_1 = AI_1 - AI$

$$= (r_1 - r)\operatorname{cosec}\frac{A}{2} = a\tan\frac{A}{2}\operatorname{cosec}\frac{A}{2} = \frac{a}{\cos(A/2)} = 8$$

33. (6): Sum of area of ΔAPD and ΔBPC is equal to 1/2 of area of parallelogram $ABCD$.

34. (4): The point E dividing $\overrightarrow{AB}$ externally in the ratio 2 : 1 is (9 , 0). Since P lies on the circle described on $\overrightarrow{DE}$ as diameter, coordinates of P are of the form $(5 + 4\cos\theta, 4\sin\theta)$

$\therefore$ maximum length of the altitude drawn from P to the base $AB = |4\sin\theta|_{\max} = 4$.

35. (1): In expansion of $(a+b)^n$ the coefficient of 2nd, 3rd and 4th term are in A.P. which gives $n = 7$

Also ${}^7C_2\, a^5 b^2 = 84$

$\Rightarrow a^5b^2 = 4$

Now $\dfrac{\log_2 a^5 + \log_2 b^2}{2} \ge (\log_2 a^5 \cdot \log_2 b^2)^{1/2}$

$\Rightarrow k \le \left(\dfrac{\log_2 a^5b^2}{2}\right)^2$, where $k = \log_2 a^5 \cdot \log_2 b^2$

$k \le 1 \Rightarrow$ maximum value of k is 1.

36. (1): $t_r = \sqrt{1+\dfrac{1}{r^2}+\dfrac{1}{(r+1)^2}} = \sqrt{\dfrac{r^2+(r+1)^2+r^2(r+1)^2}{r^2(r+1)^2}}$

$$= \sqrt{\frac{2r^2+2r+1+r^2(r^2+2r+1)}{r^2(r+1)^2}}$$

$$= \sqrt{\frac{r^4+2r^3+3r^2+2r+1}{r^2(r+1)^2}} = \frac{r^2+r+1}{r(r+1)} = \frac{1}{r(r+1)}+1$$

$$= 1+\frac{1}{r}-\frac{1}{r+1},\ S = 2000-\frac{1}{2000},\ |2000(S-2000)| = 1$$

37. (1): $\displaystyle\sum_{r=0}^{200}\alpha_r x^r = \sum_{r=0}^{200}\beta_r(1+x)^r$

$\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \ldots + \alpha_{200}x^{200}$

$= \beta_0 + \beta_1(1+x) + \ldots. + \beta_{200}(1+x)^{200}$

Equating coefficient of x^{100}, we get

$$\alpha_{100} = {}^{100}C_{100} + {}^{101}C_{100} + \ldots..\ {}^{200}C_{100} = {}^{201}C_{101}$$

Similarly we can find $\alpha_{100}, \ldots., \alpha_{200}$

$$\sum_{r=100}^{200}\alpha_r = {}^{201}C_{101} + {}^{201}C_{102} + \ldots..\ {}^{201}C_{201}$$

$A = 2^{100}(2^{101} - 1)$

When A is divided by 15 remainder is 1.

38. (0): Since $\left[\sqrt{2046}\right] = \left[\sqrt{2047}\right] = \left[\sqrt{2048}\right]$

$= \left[\sqrt{2049}\right] = 45$

$\therefore$ 2003rd term is $2003 + 45 = 2048$

Hence remainder is 0

Remark : There is no perfect square between the numbers m^2 and $(m+1)^2$.

39. (7): $f(x)+f(\alpha x)+f(\alpha^2 x)+\ldots..+f(\alpha^6 x)$

$$= 7A_0 + \sum_{k=1}^{20} A_k x^k(1+\alpha^k+\ldots\ldots+\alpha^{6k})$$

but when $k \ne 7$ and $k \ne 14$, then

$1 + \alpha^k + \alpha^{2k} + \ldots. + \alpha^{6k} = 0$

Hence $f(x) + f(\alpha x) + \ldots + f(\alpha^6 x)$

$= 7A_0 + 7A_7x^7 + 7A_{14}x^{14} = 7(A_0 + A_7x^7 + A_{14}x^{14})$

$\Rightarrow k = 7$

40. (5): Let Y_i be the subset of X such that

$Y_i = \{7m + i, m \in I\}$

$Y_0 = \{7, 14, \ldots.. 98\}, n(Y_0) = 14$

$Y_1 = \{1, 8, 15 \ldots 99\}, n(Y_1) = 15$

$Y_2 = \{2, 9, 16 \ldots , 100\}, n(Y_2) = 15$

$Y_3 = \{3, 10, 17 \ldots. 94\}, n(Y_3) = 14$

$Y_4 = \{4, 11, 18 \ldots 95\}, n(Y_4) = 14$

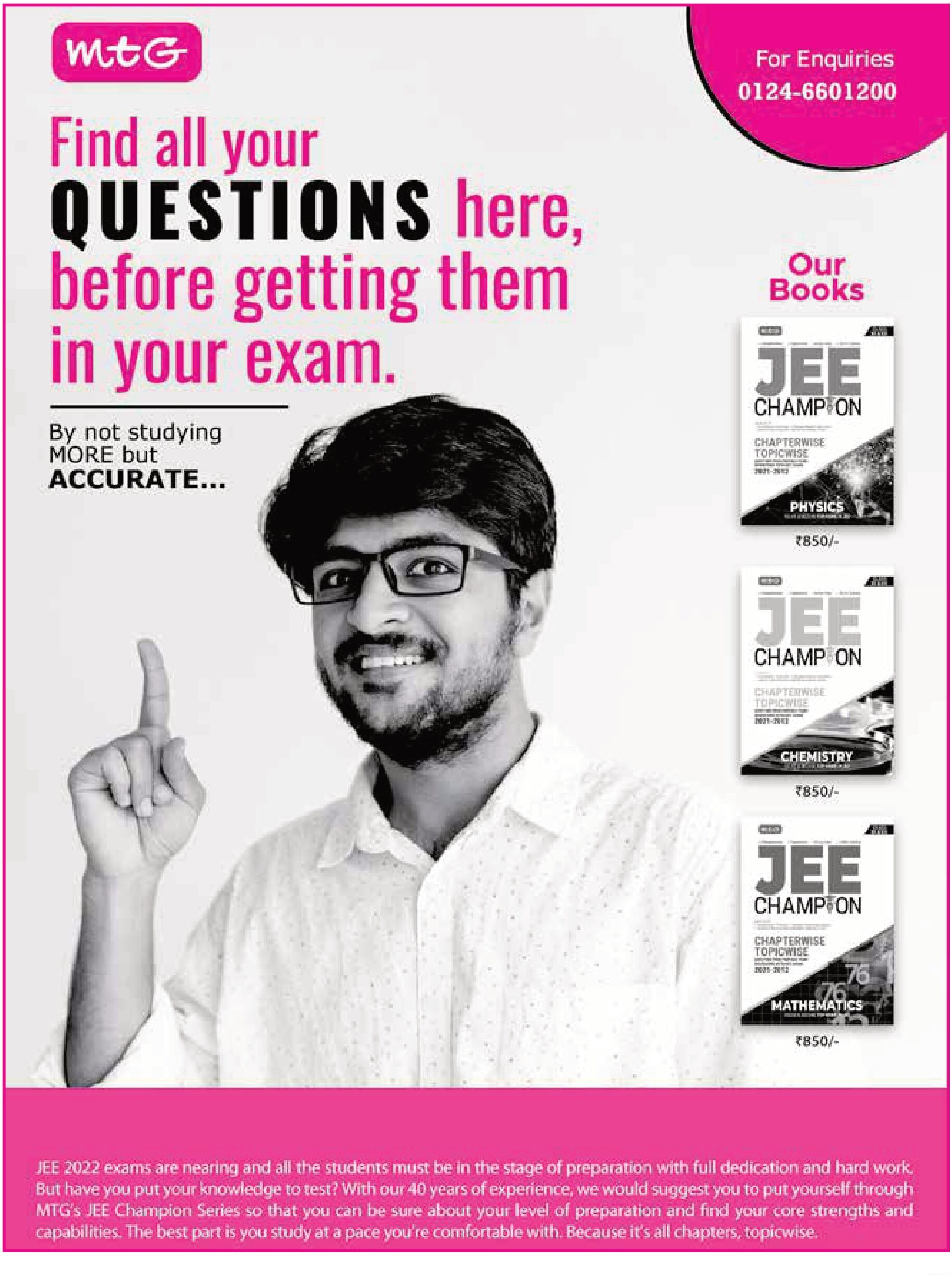
$Y_5 = \{5, 12, \ldots. 96\}, n(Y_5) = 14$

$Y_6 = \{6, 13, \ldots. 97\}, n(Y_6) = 14$

The largest Y will consist of (i) an element of Y_0 (ii) Y_1 (iii) Y_2 (iv) Y_3 or Y_4

$\Rightarrow$ The maximum possible number of elements in $Y = 1 + 15 + 15 + 14 = 45$.

⁂



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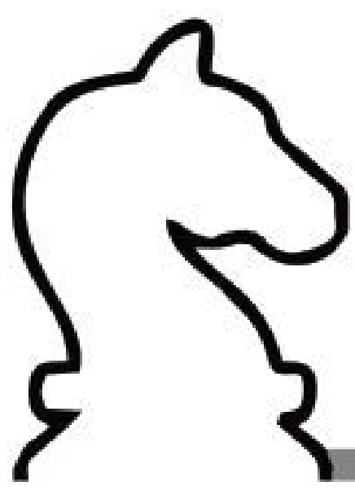

SECTION - I

Only One Correct Option

1. If $f(x+1)=\dfrac{f(x)-1}{f(x)+1}\ \forall\, x\in N$ with $f(1) = 2$, then $\displaystyle\int_1^5 \sum_{r=1}^{1728}\frac{1}{f(r)}\,dx=$

(a) 2014 (b) 2015
(c) 2016 (d) none of these

2. If $t = ax^2 + 2bx + c$, $y = at^2 + 2bt + c$, then $\displaystyle\int_{-1}^{1}\left(\frac{d^4y}{dx^4}\right)dx=$

(a) $48a^3$ (b) $24a^2$
(c) $24a^3$ (d) none of these

3. $\displaystyle\int_{-1}^{1}\left[1008+\frac{[x]}{x^{2014}+1}\right]dx=$

(where [·] denotes greatest integer function)

(a) 0 (b) 1008
(c) 2016 (d) 2015

4. If $a = 5$, $b = 6$, $A = 60°$ in ΔABC, then

(a) $\tan B=\dfrac{3\sqrt{3}}{2}$ (b) $B=\sin^{-1}\dfrac{3\sqrt{3}}{5}$
(c) $\cos B < 0$ (d) none of these

5. If α, β, γ be the angles made by a line with axes so that

$$2\left(\frac{\tan^2\alpha}{1+\tan^2\alpha}+\frac{\tan^2\beta}{1+\tan^2\beta}+\frac{\tan^2\gamma}{1+\tan^2\gamma}\right)=3\sec^2\frac{\theta}{2},$$

then $\theta=$

(a) $\pi/12$ (b) $\pi/10$
(c) $\pi/6$ (d) $\pi/3$

6. $\displaystyle\int_1^9\left(\frac{1}{2}+\sqrt{\frac{1}{4}+\log_3 t}\right)dt+\int_1^2\frac{3^{x^2-[x]}}{3^{\{x\}}}\,dx=$

(a) 80 (b) 8
(c) 17 (d) can't be determined

SECTION - II

One or More Than One Option Correct

7. If $\alpha=\displaystyle\int_0^1 e^{x^2}dx-e$, $\beta=\displaystyle\int_0^1 e^{x^2}dx-1$, then

(a) $\alpha\beta > 0$ (b) $\alpha/\beta < 0$
(c) $\beta/\alpha < 0$ (d) $\alpha\beta < 0$

8. If $\alpha=\displaystyle\int_1^3 [x]^{\{x\}}dx$ and $\beta=\displaystyle\int_1^3 \{x\}^{[x]}dx$, then

(a) α is irrational but β is rational.
(b) α is rational but β is irrational.
(c) $5\alpha - 6\beta$ is irrational.
(d) $\dfrac{\alpha-1}{\alpha-\beta}$ is irrational.

9. $\displaystyle\int_0^{\pi/2} e^{\sqrt{\sin x}}\cdot\sin 2x\,dx$ belongs to

(a) $(2, e)$ (b) $(e - 1, \pi - 1)$
(c) (e, π) (d) $[1, e]$

10. If $\displaystyle\frac{2014\alpha+2015}{1007\alpha}\int_0^{1007}\left(\left\{\sin\frac{\pi x}{2014}\right\}+\left\{\sin\left(\frac{-\pi x}{2014}\right)\right\}\right)dx=2015$

(where $\{x\}$ denotes fractional part function},
then $\alpha=$

(a) 1007 (b) 2014
(c) 2015 (d) none of these

11. The digit in unit place of $2^{(9^{100}-1)}$ is divisible by
(a) 2 (b) 4 (c) 3 (d) 6

12. If the cubic equation $x^3 + k = 3x$ has all integral roots, then
(a) $|k|$ is prime.
(b) $[k]$ satisfy $x^2 - 4 = 0$.
(c) $|[k]|$ can be eccentricity of an ellipse.
(d) k^2 is the area of the triangle formed by $x - 2y = 4$ with axes.

13. If z_1, z_2, z_3 be the vertices of an equilateral triangle ABC and satisfy $z^3 + 3\alpha z + \beta = 3\gamma z^2$, then equation of tangent to $x = y^2$ at $P(\alpha^2, \gamma^2)$ will be
(a) $x - 2\alpha y + \alpha^2 = 0$ (b) $x = \gamma^2(2y - \alpha)$
(c) $x + \gamma^4 = 2\alpha y$ (d) $x - 2\alpha y + \beta = 0$

14. If $f(x)$ be a non-negative function so that $|41 + f(x) - 4x^2| - 5 = |f(x)| + 4|9 - x^2|$ is true for $x \in [\alpha, \beta]$, then $\int\limits_{\alpha}^{\beta}(ax^{\beta} + bx^{\beta-2} + cx^{\alpha+\beta})dx$ will not depend on
(a) a (b) b (c) c (d) a, b, c

15. If $f''(x)$ is continuous $\forall\ x \in R$ and $f(0) = 1 = f'(1)$ such that $\int\limits_{0}^{f(0)} zf''(z)dz$ vanishes, then
(a) $f(1) = f(0) + f'(1)$
(b) $f'(1) = \sum\limits_{r=1}^{\infty}\dfrac{1}{\{f(1)\}^r}$
(c) $f(0), f(1), f'(1)$ are in A.P.
(d) $\sin^{-1}(\sin(f(1)))$ is defined

SOLUTIONS

1. **(c):** $f(1) = 2$; $f(2) = \dfrac{2-1}{2+1} = \dfrac{1}{3}$;

$$f(3) = \frac{\frac{1}{3}-1}{\frac{1}{3}+1} = -\frac{1}{2};\ f(4) = \frac{-\frac{1}{2}-1}{-\frac{1}{2}+1} = -3;$$

$$f(5) = \frac{-3-1}{-3+1} = 2;\ f(6) = \frac{2-1}{2+1} = \frac{1}{3} \text{ etc.}$$

$\therefore$ $f(1) = f(5) = f(9) = \ldots = 2$
$f(2) = f(6) = \ldots = 1/3$; $f(3) = f(7) = \ldots = -1/2$
$f(4) = f(8) = \ldots = -3$

The above result shows that $f(x)$ is periodic with period 4.

Now, $\dfrac{1}{f(1)} + \dfrac{1}{f(2)} + \dfrac{1}{f(3)} + \dfrac{1}{f(4)} = \dfrac{1}{2} + 3 - 2 - \dfrac{1}{3} = \dfrac{7}{6}$

$$\therefore \int\limits_1^5 \sum_{r=1}^{1728}\frac{1}{f(r)}dx$$

$$= \int\limits_1^5\left[\frac{1}{f(1)} + \frac{1}{f(2)} + \frac{1}{f(3)} + \frac{1}{f(4)} + \frac{1}{f(1)} + \frac{1}{f(2)} + \ldots\text{to 1728 terms}\right]dx$$

$$= \int\limits_1^5 432\left\{\frac{1}{f(1)} + \frac{1}{f(2)} + \frac{1}{f(3)} + \frac{1}{f(4)}\right\}dx$$

$$\left[\because \frac{1728}{4} = 432\right]$$

$$= 432\int\limits_1^5 \frac{7}{6}dx = 504\left[x\right]_1^5 = 2016$$

2. **(a) :** $\dfrac{dt}{dx} = 2ax + 2b$; $\dfrac{dy}{dt} = 2at + 2b$

$$\therefore \frac{dy}{dx} = 4(ax+b)(at+b)$$

$$\Rightarrow \frac{d^2y}{dx^2} = 4\left[a(at+b) + (ax+b)\cdot a\cdot\frac{dt}{dx}\right]$$

$$= 4a\left[(at+b) + (ax+b)\cdot 2(ax+b)\right]$$

$$= 4a[(at + b) + 2(ax + b)^2]$$

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For MTG Learning Media Pvt. Ltd.
Mahabir Singh
Director

$$\Rightarrow \frac{d^3y}{dx^3} = 4a\left[a\cdot\frac{dt}{dx} + 2\cdot 2(ax+b)a\right]$$

$$= 4a^2[2(ax+b) + 4(ax+b)] = 24a^2(ax+b)$$

Now, $\int\limits_{-1}^{1}\left(\frac{d^4y}{dx^4}\right)dx = \left[\frac{d^3y}{dx^3}\right]_{-1}^{1}$

$$= 24a^2\left[ax+b\right]_{-1}^{1} = 24a^2\cdot 2a = 48a^3$$

3. (d) : If $-1 \le x < 0$, $[x] = -1$ and $1 < x^{2014} + 1 \le 2$

$$\Rightarrow \frac{1}{2} \le \frac{1}{x^{2014}+1} < 1 \Rightarrow -\frac{1}{2} \ge \frac{-1}{x^{2014}+1} > -1$$

$$\Rightarrow -\frac{1}{2} \ge \frac{[x]}{x^{2014}+1} > -1 \quad [\because [x] = -1]$$

$$\Rightarrow 1007\frac{1}{2} \ge 1008 + \frac{[x]}{x^{2014}+1} > 1007$$

$$\therefore \left[1008 + \frac{[x]}{x^{2014}+1}\right] = 1007$$

If $0 \le x < 1$, $[x] = 0$ and so $\frac{[x]}{x^{2014}+1} = 0$

$$\Rightarrow 1008 + \frac{[x]}{x^{2014}+1} = 1008$$

$$\therefore \left[1008 + \frac{[x]}{x^{2014}+1}\right] = 1008$$

$$\therefore I = \int\limits_{-1}^{0} 1007\,dx + \int\limits_{0}^{1} 1008\,dx = 2015$$

4. (d) : Using $\frac{a}{\sin A} = \frac{b}{\sin B}$, we have

$$\frac{5}{(\sqrt{3}/2)} = \frac{6}{\sin B} \Rightarrow \sin B = \frac{3\sqrt{3}}{5}(\text{Absurd})$$

5. (d) : We know that,

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow \Sigma\cos^2\alpha = 1$ ... (i)

Given that

$$2\left(\frac{\tan^2\alpha}{1+\tan^2\alpha} + \frac{\tan^2\beta}{1+\tan^2\beta} + \frac{\tan^2\gamma}{1+\tan^2\gamma}\right) = 3\sec^2\frac{\theta}{2}$$

$$\Rightarrow 2(\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 3\sec^2\frac{\theta}{2}$$

$$\Rightarrow 2(1-\cos^2\alpha + 1 - \cos^2\beta + 1 - \cos^2\gamma) = 3\sec^2\frac{\theta}{2}$$

$$\Rightarrow 2(3 - \Sigma\cos^2\alpha) = 3\sec^2\frac{\theta}{2} \Rightarrow 4\cos^2\frac{\theta}{2} = 3$$

[From (i)]

$$\Rightarrow 2(1+\cos\theta) = 3 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

6. (c): Let $y = f(x) = \frac{3^{x^2-[x]}}{3^{\{x\}}}$

$\therefore f(x) = 3^{x^2-([x]+\{x\})} = 3^{x^2-x} = y$ ... (i)

$\therefore x^2 - x - \log_3 y = 0$

$$\Rightarrow x = \frac{1 \pm \sqrt{1 + 4\log_3 y}}{2} = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \log_3 y}$$

$\therefore x = \frac{1}{2} + \sqrt{\frac{1}{4} + \log_3 y}$ as $y \ge 1$ if $x \ge 1$ [From (i)]

$$\therefore f^{-1}(x) = \frac{1}{2} + \sqrt{\frac{1}{4} + \log_3 x}$$

Also, $f(1) = 1$ and $f(2) = 9$

$$\therefore I = \int\limits_{1}^{9}\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \log_3 t}\right)dt + \int\limits_{1}^{2} 3^{x^2-x}\,dx$$

$$= \int\limits_{1}^{9} f^{-1}(t)dt + \int\limits_{1}^{2} 3^{t^2-t}\cdot dt = \int\limits_{f(1)}^{f(2)} f^{-1}(t)dt + \int\limits_{1}^{2} f(t)dt$$

$= 2f(2) - 1\,f(1) = 2(9) - 1(1) = 17$

Note : $\int\limits_{a}^{b} f(x)dx + \int\limits_{f(a)}^{f(b)} f^{-1}(x)dx = bf(b) - af(a)$

PUZZLE CORNER

SOLUTION · FEBRUARY 2022

3÷ 2	6	2– 3	1	1– 4	5
12× 4	1– 2	7+ 1	6	8+ 5	3
1	3	1– 4	5	3÷ 6	2
3	6+ 1	5	2÷ 4	1– 2	5– 6
1– 5	2– 4	6	2	3	1
6	10+ 5	2	3	5+ 1	4

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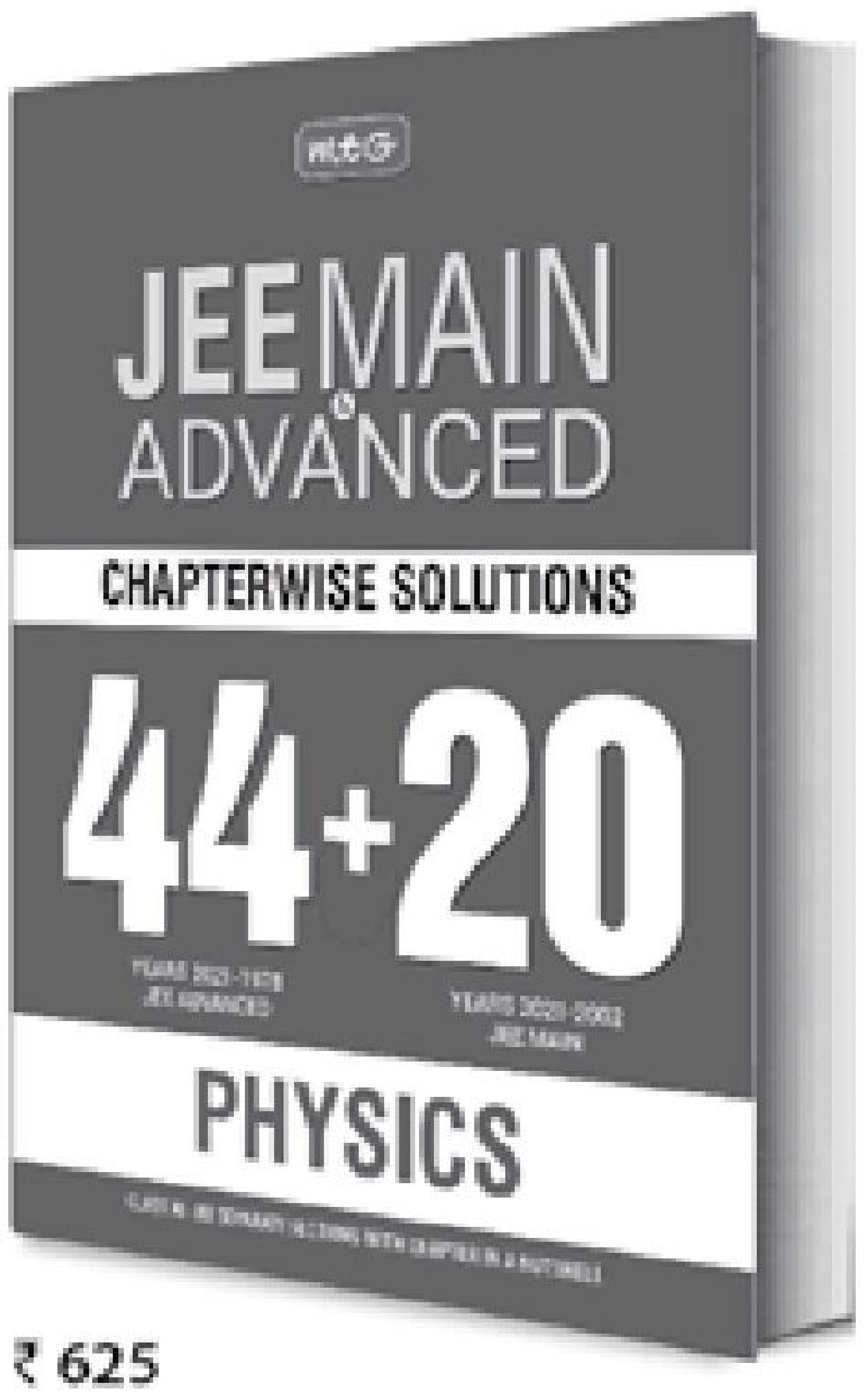
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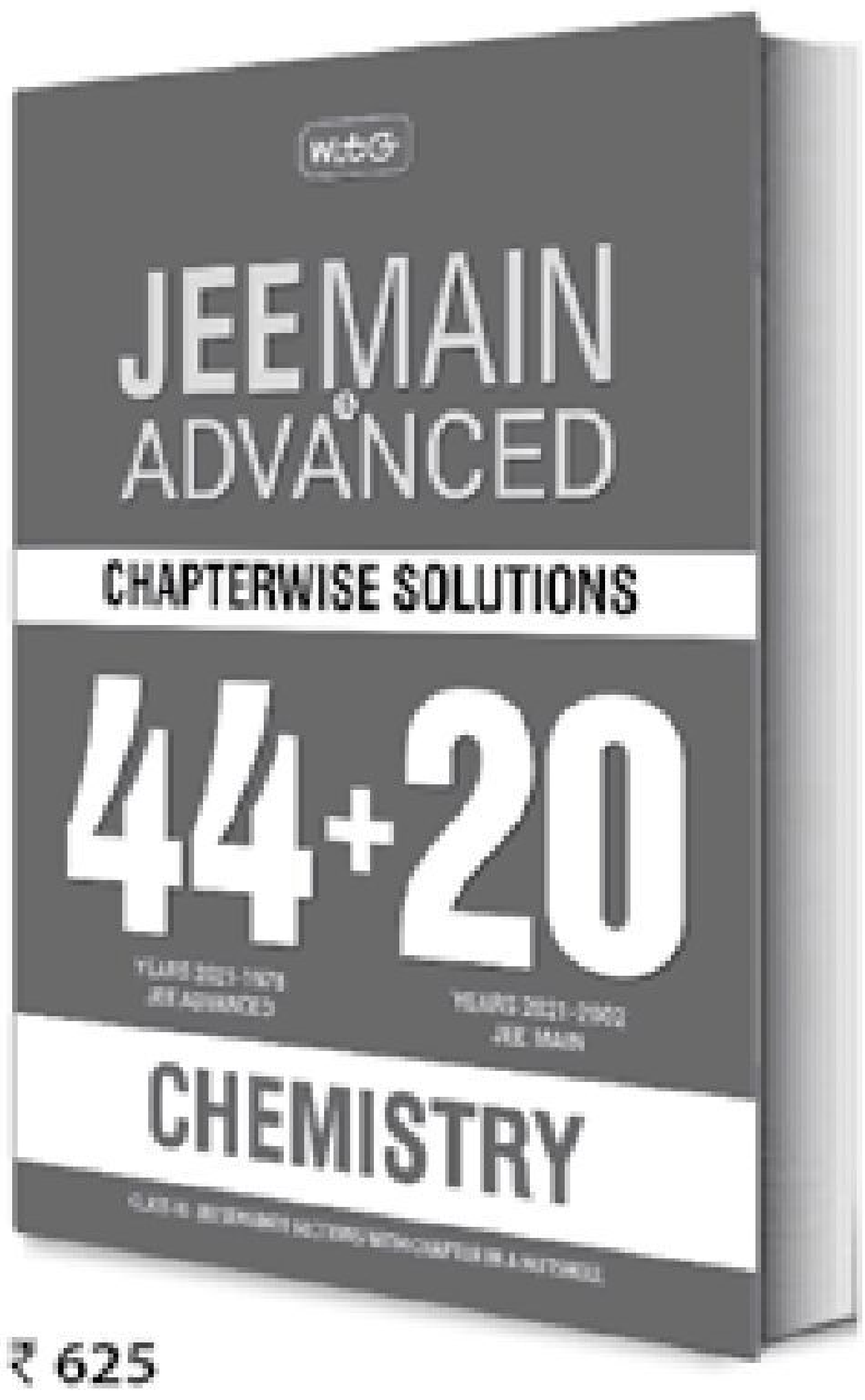
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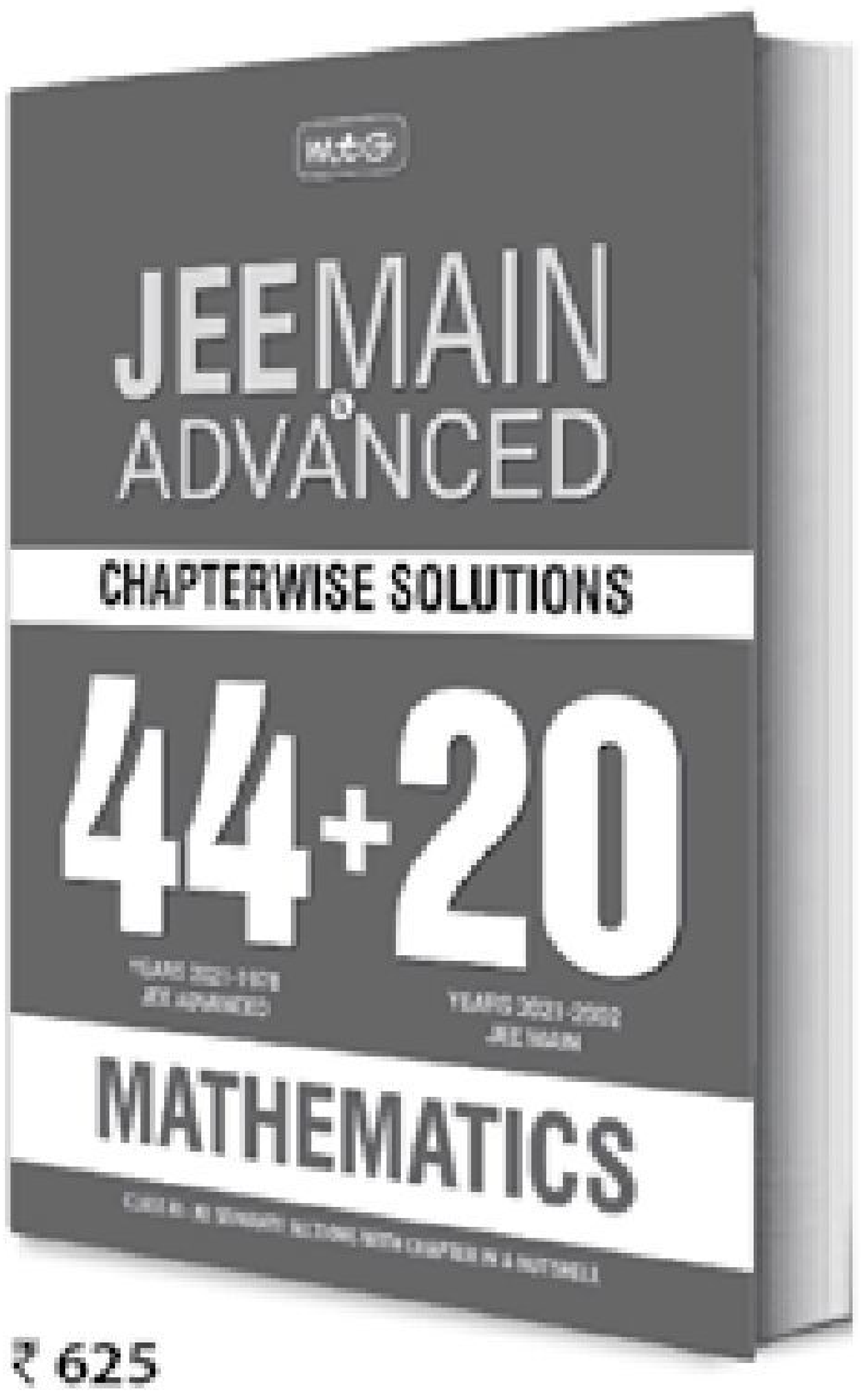
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7. (b, c, d) : In (0, 1), $x^2 < 1 \Rightarrow e^{x^2} < e^1$

$$\Rightarrow \int_0^1 e^{x^2}dx < \int_0^1 e\,dx = \int_0^1 e^{x^2}dx - e < 0 \Rightarrow \alpha < 0$$

Also, $x^2 > 0$ in (0, 1)

$$\Rightarrow e^{x^2} > e^0 \Rightarrow \int_0^1 e^{x^2}dx > \int_0^1 e^0 dx$$

$$\Rightarrow \int_0^1 e^{x^2}dx - 1 > 0 \Rightarrow \beta > 0$$

$$\therefore \alpha\beta < 0, \frac{\alpha}{\beta} < 0, \frac{\beta}{\alpha} < 0$$

8. (a, c, d) : $\alpha = \int_1^2 1^{x-1}dx + \int_2^3 2^{x-2}dx = 1 + \frac{1}{\log_e 2}$

which is irrational.

And, $\beta = \int_1^2 (x-1)^1 dx + \int_2^3 (x-2)^2 dx$

$$= \left[\frac{x^2}{2} - x\right]_1^2 + \left[\frac{(x-2)^3}{3}\right]_2^3 = \frac{1}{2} + \left(\frac{1}{3} - 0\right) = \frac{5}{6}$$

which is rational.

$\therefore 5\alpha - 6\beta = \frac{5}{\log_e 2}$, which is irrational.

Also, $\frac{\alpha - 1}{\alpha - \beta}$ is also irrational.

9. (a, d) : $I = \int_0^1 e^z \cdot 2z^2 \cdot 2z\,dz$

Put $\sin x = z^2$, $\cos x\,dx = 2z\,dz$

$$= 4\left[(z^3 - 3z^2 + 6z - 6)e^z\right]_0^1$$

x	0	$\pi/2$
z	0	1

$= 4[-2e + 6] = 4(6 - 5.436564)$ (app.)

[$\because e = 2.718282$ (app.)]

$= 2.253744$ (app.) $\therefore$ (a) and (d) are correct.

10. (c): $\because$ In (0, 1007), $\sin\frac{\pi x}{2014} \in (0, 1)$

and so, $\sin\frac{\pi x}{2014}$ is not an integer.

Since, $\{x\} + \{-x\} = 1$, if $x \notin I$

$$\therefore \left\{\sin\frac{\pi x}{2014}\right\} + \left\{\sin\left(\frac{-\pi x}{2014}\right)\right\} = \left\{\sin\frac{\pi x}{2014}\right\} + \left\{-\sin\frac{\pi x}{2014}\right\} = 1$$

$\therefore$ We have,

$$\frac{2014\alpha + 2015}{1007\alpha}\int_0^{1007}\left(\left\{\sin\frac{\pi x}{2014}\right\} + \left\{\sin\left(\frac{-\pi x}{2014}\right)\right\}\right)dx = 2015$$

gives $\frac{2014\alpha + 2015}{1007\alpha}\int_0^{1007} 1\,dx = 2015$

$$\Rightarrow \frac{2014\alpha + 2015}{\alpha} = 2015 \Rightarrow \alpha = 2015$$

11. (a, c, d) : $9^{100} - 1 = (1 + 8)^{100} - 1$

$= {}^{100}C_0 \cdot 8 + {}^{100}C_2 \cdot 8^2 + \dots = 8n$ (say)

Also, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$ and any integral power of 16 always have 6 at its unit place.

$\therefore 2^{(9^{100} - 1)} = 2^{8n} = 16^{2n}$ will have 6 at its units place.

12. (a, b, d) : Let α, β, γ be the integral roots of $x^3 + k = 3x$

$\therefore \Sigma\alpha = 0$ ... (1)

$\Sigma\alpha\beta = -3$ ... (2)

$\alpha\beta\gamma = -k$ ... (3)

$\because \Sigma\alpha^2 = (\Sigma\alpha)^2 - 2\Sigma\alpha\beta = 6$

$\Rightarrow |\alpha|, |\beta|, |\gamma| \in \{0, 1, 2\}$

$\therefore \Sigma\alpha = 0$ and $\Sigma\alpha^2 = 6$

$\Rightarrow \alpha = 1, \beta = 1, \gamma = -2$ or $\alpha = -1, \beta = -1, \gamma = 2$

Only for these values, (2) is satisfied.

$\therefore$ (3) gives $k = \pm 2 \Rightarrow |k| = 2$ and $[k] = \pm 2$

$\Rightarrow |k| = 2$, which is prime; $[k]$ satisfy $x^2 - 4 = 0$

$\because |[k]| = 2$, which can't be eccentricity of any ellipse (as $e < 1$).

Also, $x - 2y = 4$ *i.e.* $\frac{x}{4} + \frac{y}{-2} = 1$ forms an area $= 4 = k^2$ with axes.

13. (a, b, c) : $z^3 - 3\gamma z^2 + 3\alpha z + \beta = 0$

Its roots are z_1, z_2 and z_3

$\therefore z_1 + z_2 + z_3 = 3\gamma$; $z_1z_2 + z_2z_3 + z_3z_1 = 3\alpha$

and $z_1z_2z_3 = -\beta$

For equilateral ΔABC, we know that $\Sigma z_1^2 = \Sigma z_1z_2$

$\Rightarrow (\Sigma z_1)^2 - 2\Sigma z_1z_2 = \Sigma z_1z_2$

$\Rightarrow (3\gamma)^2 = 3(3\alpha) \Rightarrow \alpha = \gamma^2$

From $x = y^2$, we have, $1 = 2y \cdot \frac{dy}{dx}$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{\text{at } P(\alpha^2, \gamma^2)} = \frac{1}{2\gamma^2} = \frac{1}{2\alpha}$$

$\therefore$ Equation of tangent to $x = y^2$ at $P(\alpha^2, \gamma^2)$ will be

$$y-\gamma^2=\frac{1}{2\alpha}(x-\alpha^2)$$

$\Rightarrow$ $2\alpha y - 2\alpha^2 = x - \alpha^2$ [Since $\gamma^2 = \alpha$]

$\Rightarrow$ $x - 2\alpha y + \alpha^2 = 0$ or $x = \gamma^2(2y - \alpha)$

or $x + \gamma^4 = 2\alpha y$

14. (a, b): $\because$ $41 + f(x) - 4x^2 = f(x) + (36 - 4x^2) + 5$

$\therefore$ $|41 + f(x) - 4x^2| = |f(x) + (36 - 4x^2) + 5|$

$$\le |f(x)| + |36 - 4x^2| + |5|$$

$\Rightarrow$ $|41 + f(x) - 4x^2| \le |f(x)| + 4|9 - x^2| + 5$

Equality will hold only if $f(x)\cdot(9 - x^2) \ge 0$ [$\because$ $5 > 0$]

i.e. $|41 + f(x) - 4x^2| - 5 = |f(x)| + 4|9 - x^2|$ only if $f(x)(9 - x^2) \ge 0$

$\Rightarrow$ $9 - x^2 \ge 0$ [Since $f(x) \nless 0$]

$\Rightarrow$ $x^2 - 9 \le 0$ $\Rightarrow$ $-3 \le x \le 3$

$\Rightarrow$ $x \in [-3, 3]$ $\Rightarrow$ $\alpha = -3, \beta = 3$

Now, $\displaystyle\int_{\alpha}^{\beta}(ax^{\beta}+bx^{\beta-2}+cx^{\alpha+\beta})dx$

$$=\int_{-3}^{3}(ax^3+bx+c)dx=\int_{-3}^{3}c\,dx$$

[$\because$ $ax^3 + bx$ is an odd function]

$= 6c$, which does not depend on *a* and *b*.

15. (a, b, d) : Since $f(0) = 1 = f'(1)$ ... (1)

And, $\displaystyle\int_{0}^{f(0)} zf''(z)dz$ vanishes

$$\therefore \int_0^1 zf''(z)dz=0 \Rightarrow \left[zf'(z)\right]_0^1-\int_0^1 1\cdot f'(z)dz=0$$

$$\Rightarrow f'(1)-\left[f(z)\right]_0^1=0 \Rightarrow f'(1)-f(1)+f(0)=0$$

$\Rightarrow$ $1 - f(1) + 1 = 0$ [Using (1)]

$\Rightarrow$ $f(1) = 2$ $\therefore$ $f(1) = f(0) + f'(1)$

$$\sum_{r=1}^{\infty}\frac{1}{\{f(1)\}^r}=\frac{1}{f(1)}+\frac{1}{\{f(1)\}^2}+\frac{1}{\{f(1)\}^3}+\ldots \text{ to } \infty$$

$$=\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots\ldots=\frac{1/2}{1-\frac{1}{2}}=1=f'(1)$$

$$\therefore\ f'(1)=\sum_{r=1}^{\infty}\frac{1}{\{f(1)\}^r}$$

$f(0), f(1), f'(1)$ are not in A.P.

$\sin^{-1}(\sin f(1)) = \sin^{-1}(\sin 2) = \pi - 2 \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore$ $\sin^{-1}(\sin f(1))$ is defined.

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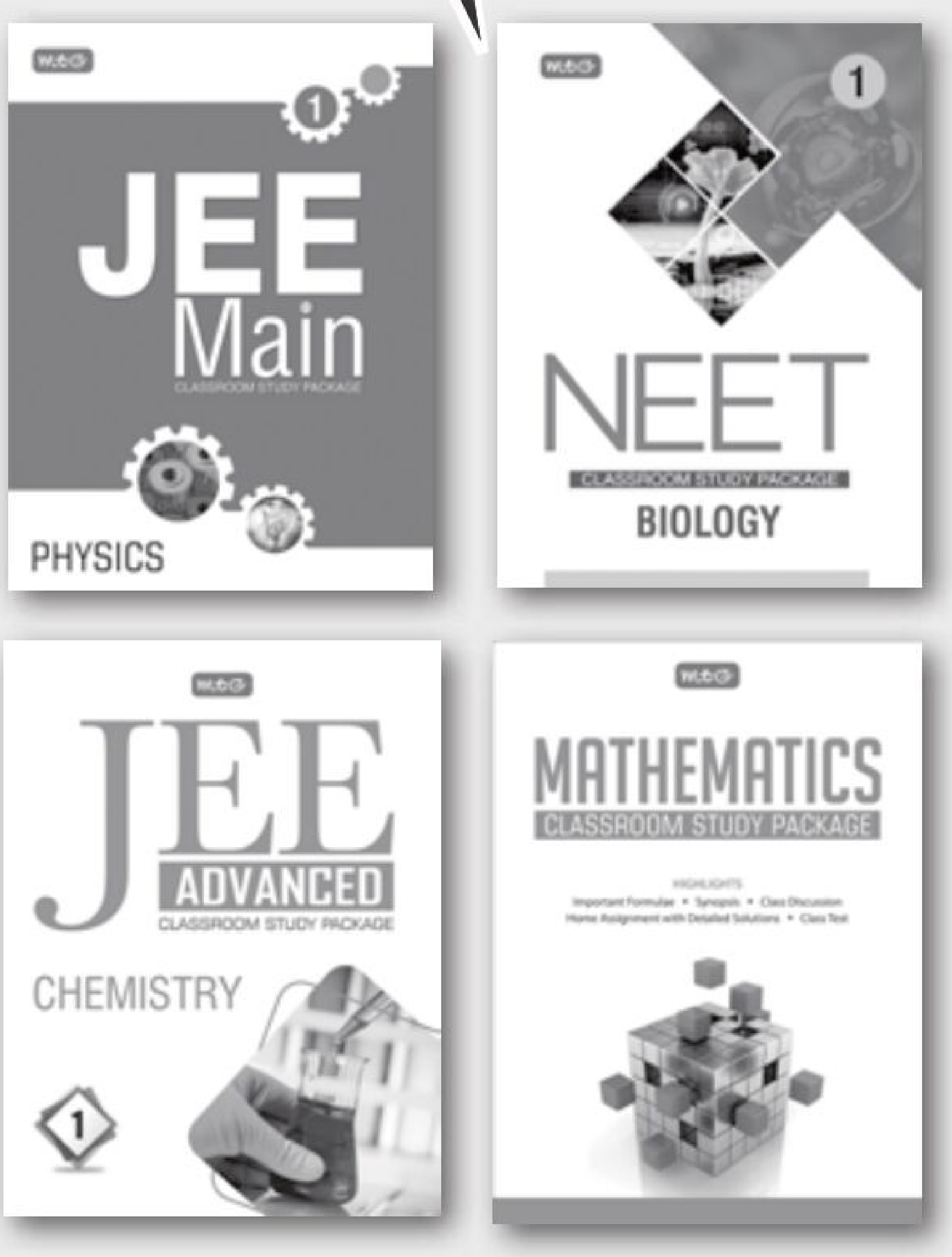

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CONCEPT MAP

PERMUTATIONS AND COMBINATIONS

Class XI

Properties

- ${}^nP_n = n! = n(n-1)\cdot(n-2)\ldots 3\cdot2\cdot1 = {}^nP_{n-1}$
- ${}^nP_1 = n$
- ${}^nP_0 = \dfrac{n!}{(n-0)!} = 1$
- ${}^nP_r = n\cdot{}^{n-1}P_{r-1} = n(n-1)\,{}^{n-2}P_{r-2}$
  $= n(n-1)(n-2)\,{}^{n-3}P_{r-3}$ and so on
- ${}^{n-1}P_r + r\cdot{}^{n-1}P_{r-1} = {}^nP_r$
- $\dfrac{{}^nP_r}{{}^nP_{r-1}} = n-r+1$

Restricted Permutations

The number of ways in which r objects can be arranged from n dissimilar objects if k particular objects are

- Always included (or never excluded)
  $= {}^{n-k}C_{r-k}\,r! = {}^rP_k\,{}^{n-k}P_{r-k}$
- Always excluded (never included)
  $= {}^{n-k}C_r\,r! = {}^{n-k}P_r$

Permutations

Arranging r objects out of n different things

- When repetition is not allowed $= {}^nP_r = \dfrac{n!}{(n-r)!}$, where $0 \le r \le n$
- When repetition is allowed $= n^r$

Distributions into Groups

Distribution of n distinct things into r groups $G_1, G_2, \ldots, G_r$ containing $P_1, P_2, \ldots, P_r$ elements respectively.

- Groups are distinct : $\dfrac{n!}{P_1!P_2!\ldots P_r!}r!$
- Groups are identical : $\dfrac{n!}{P_1!P_2!\ldots P_r!}$

Circular Permutations

(i) Arrangement of n different things taken all at a time in form of circle

$= (n-1)!$, if sense matter.

$= 1/2(n-1)!$, if sense doesn't matter

(ii) Number of circular permutations of n dissimilar things taken r at a time

$= \dfrac{{}^nP_r}{r}$, if clockwise and anticlockwise orders are considered as different.

$= \dfrac{{}^nP_r}{2r}$, if clockwise and anticlockwise order is considered as same.

PERMUTATIONS AND COMBINATIONS

De-arrangements

Any change in the existing order of things is called De-arrangement. If m things are arranged in a row, the number of ways in which they can be dearranged so that none of them occupies its original place (no one of them occupies the place assigned to it)

$$= m!\left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots + (-1)^m\frac{1}{m!}\right]$$

$$= m!\sum_{r=0}^{m}(-1)^r\frac{1}{r!}, \text{ we denote it by } D(m)$$

$$= {}^mP_m - {}^mP_{m-1} + {}^mP_{m-2} - \ldots + (-1)^m\,{}^mP_0$$

Factorial Notation

Product of first n natural numbers is denoted by $n!$

i.e., $n! = n(n-1)(n-2)\ldots 3\cdot2\cdot1$

Fundamental Principle of Counting

In an operation A can be performed in m different ways and another operation B can be performed in n different ways, then

- Both the operations can be performed in $m \times n$ ways.
- Either of the two operations can be performed in $(m + n)$ ways.

Combinations

- Selecting r objects out of n different things given by ${}^nC_r = \dfrac{n!}{r!(n-r)!}, 0 \le r \le n$

Properties

- ${}^nP_r = {}^nC_r\,r!, 0 \le r \le n$
- For $0 \le r \le n$, ${}^nC_r = {}^nC_{n-r}$
- For $1 \le r \le n$, ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- ${}^nC_a = {}^nC_b \Rightarrow a = b$ or $n = a + b$
- ${}^nC_0 + {}^nC_1 + \ldots + {}^nC_n = 2^n$

Restricted Combinations

The number of ways in which r objects can be selected from n dissimilar objects if k particular objects are

- Always included $= {}^{n-k}C_{r-k} = {}^{n-k}C_{n-r}$
- never included (Always excluded) $= {}^{n-k}C_r$

DEFINITE INTEGRALS AND APPLICATIONS OF INTEGRALS

Class XII

CONCEPT MAP

Fundamental Theorem of Calculus

- **First Fundamental Theorem :** Let $f(x)$ be a continuous function on the closed interval $[a, b]$ and let $A(x)$ be the area function. Then $A'(x) = f(x)$, for all $x \in [a, b]$.
- **Second Fundamental Theorem :** Let $f(x)$ be a continuous function on the closed interval $[a, b]$ and $F(x)$ be an integral of $f(x)$, then $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

Properties

- $\int_a^b f(x)dx = \int_a^b f(t)dt$
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$
  In particular $\int_a^a f(x)dx = 0$
- $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$,
  where $a < c < b$
- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
- $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
- $\int_{-a}^a f(x)dx = \begin{cases} 0 & ,\text{if } f(-x) = -f(x) \\ 2\int_0^a f(x)dx & ,\text{if } f(-x) = f(x) \end{cases}$
- $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$
- $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$

Solving by Substitution

When definite integral is to be found by substitution, change the lower and upper limits of integration. If substitution is $t = f(x)$ and lower limit of integration is a and upper limit is b, then new lower and upper limits will be $f(a)$ and $f(b)$ respectively.

For any two values a and b, we have $\int_a^b f(x)\,dx = [F(x) + c]_a^b$
$= F(b) - F(a)$

Here, $F(x)$ is anti derivative of function $f(x)$.

DEFINITE INTEGRALS

Limit as a Sum

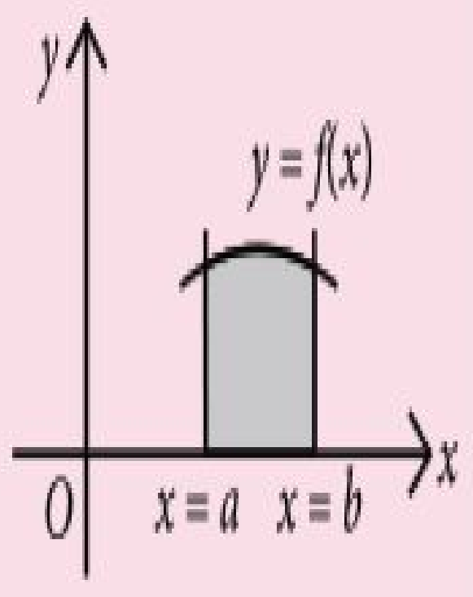
$$\int_a^b f(x)\,dx = \lim_{h\to0} h[f(a) + f(a+h) + \ldots + f(a + (n-1)h)],$$

where $h = \dfrac{b-a}{n} \to 0$ as $n \to \infty$

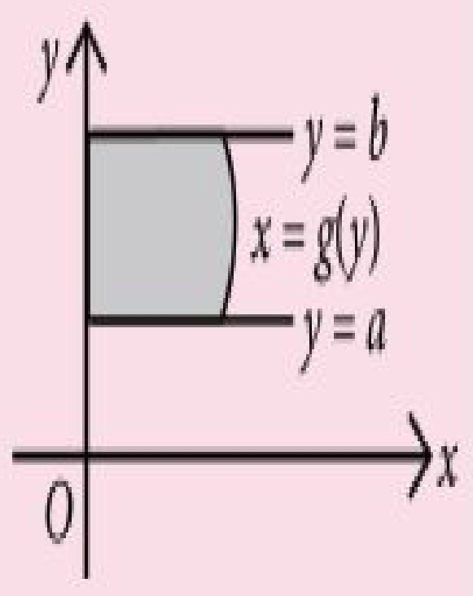
APPLICATIONS OF INTEGRALS

Area Under Simple Curve

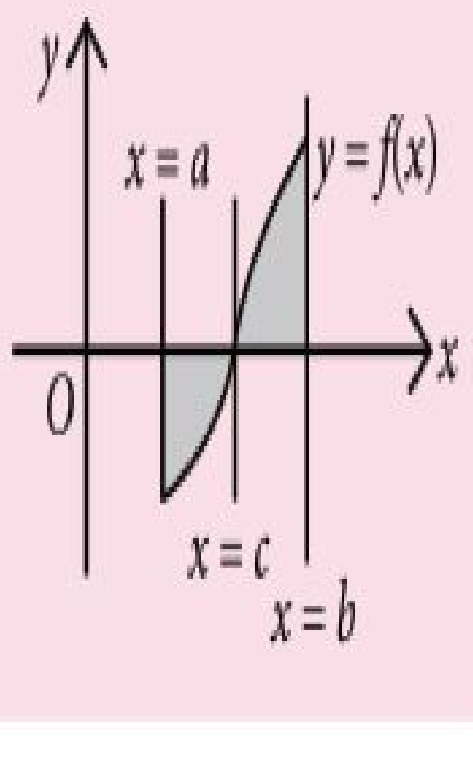
- Area $= \int_a^b y\,dx$
  $= \int_a^b f(x)dx$ (where $b > a$)



- Area $= \int_a^b x\,dy$
  $= \int_a^b g(y)dy$ (where $b > a$)

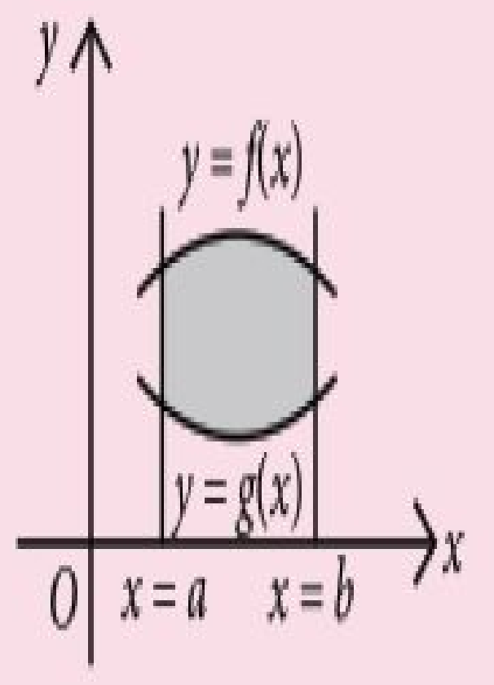


- Area $= \left|\int_a^c f(x)dx\right| + \int_c^b f(x)dx$

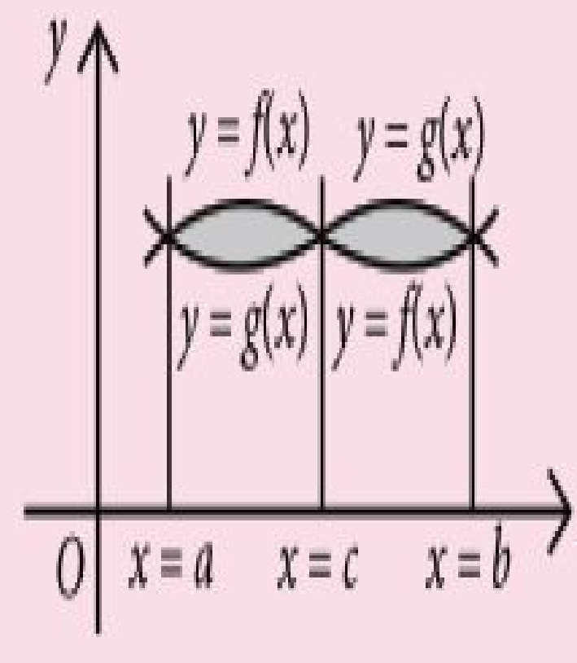


Area Between Two Curves

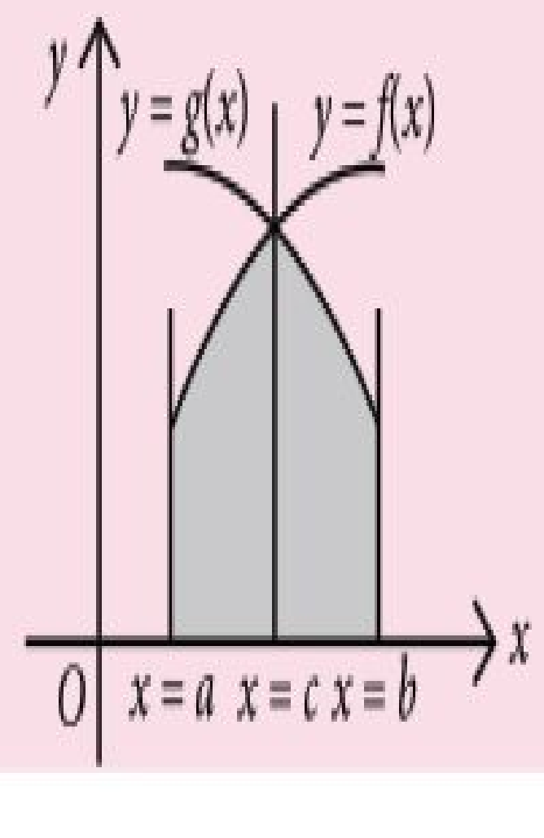
- Area $= \int_a^b [f(x) - g(x)]dx$,
  $f(x) \ge g(x)$ in $[a, b]$



- Area $= \int_a^c [f(x) - g(x)]dx + \int_c^b [g(x) - f(x)]dx$,
  where $f(x) \ge g(x)$ in $[a, c]$ and $f(x) \le g(x)$ in $[c, b]$



- Area $= \int_a^c f(x)dx + \int_c^b g(x)dx$



JEE 2022
PRACTICE PAPER
ADVANCED

Time : 1 hr.

Series - 4

Topics to be covered : Differential Calculus / Integral Calculus / Vector Algebra

PAPER-1

SECTION 1

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options. (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*

1. The value of $\lim\limits_{h\to 0} 2\left\{\dfrac{\sqrt{3}\sin\left(\dfrac{\pi}{6}+h\right)-\cos\left(\dfrac{\pi}{6}+h\right)}{\sqrt{3}h(\sqrt{3}\cos h-\sin h)}\right\}$ is

(a) 3/4 (b) 4/3 (c) 2/3 (d) $2/\sqrt{3}$

2. Let $f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\dfrac{1-x}{x}}\right)\right)$, $0 < x < 1$. Then

(a) $(1 + x)^2 f'(x) + 2(f(x))^2 = 0$

(b) $(1 - x)^2 f'(x) + 2(f(x))^2 = 0$

(c) $(1 - x)^2 f'(x) - 2(f(x))^2 = 0$

(d) $(1 + x)^2 f'(x) - 2(f(x))^2 = 0$

3. The value of the integral

$$\int \frac{\sin\theta\cdot\sin 2\theta(\sin^6\theta+\sin^4\theta+\sin^2\theta)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}\,d\theta$$

is (where c is a constant of integration)

(a) $\dfrac{1}{18}[9-2\sin^6\theta-3\sin^4\theta-6\sin^2\theta]^{3/2}+c$

(b) $\dfrac{1}{18}[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta]^{3/2}+c$

(c) $\dfrac{1}{18}[9-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta]^{3/2}+c$

(d) $\dfrac{1}{18}[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta]^{3/2}+c$

4. Let three vectors $\vec{a},\vec{b}$ and $\vec{c}$ be such that $\vec{a}\times\vec{b}=\vec{c}$, $\vec{b}\times\vec{c}=\vec{a}$ and $|\vec{a}|=2$. Then, which one of the following is not true?

(a) $[\vec{a}\,\vec{b}\,\vec{c}]+[\vec{c}\,\vec{a}\,\vec{b}]=8$

(b) Projection of $\vec{a}$ on $(\vec{b}\times\vec{c})$ is 2.

(c) $|3\vec{a}+\vec{b}-2\vec{c}|^2=51$ (d) $\vec{a}\times((\vec{b}+\vec{c})\times(\vec{b}-\vec{c}))=\vec{0}$

SECTION 2

- *This section contains THREE (03) questions stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*

Question Stem for Question Nos. 5 and 6

Consider the function $f(x)$, a fourth degree polynomial such that $\lim\limits_{x\to 1}\dfrac{f(x)}{(x-1)^2} = 1$, $f'(0) = -6$, $f'(2) = 6$.

5. The minimum value of $f(x)$ is ______.

6. The length of subtangent of the curve $y = f(x)$ where it cuts the y-axis is $\dfrac{1}{\lambda}$, then $\lambda =$ ______.

Question Stem for Question Nos. 7 and 8

For integral $\int f\left(x-\dfrac{a}{x}\right)\cdot\left(1+\dfrac{a}{x^2}\right)dx$, put $x-\dfrac{a}{x}=t$

For integral $\int f\left(x+\dfrac{a}{x}\right)\cdot\left(1-\dfrac{a}{x^2}\right)dx$, put $x+\dfrac{a}{x}=t$

For integral $\int f\left(x^2-\dfrac{a}{x^2}\right)\cdot\left(x+\dfrac{a}{x^3}\right)dx$, put $x^2-\dfrac{a}{x^2}=t$

For integral $\int f\left(x^2+\dfrac{a}{x^2}\right)\cdot\left(x-\dfrac{a}{x^3}\right)dx$, put $x^2+\dfrac{a}{x^2}=t$

many integrands can be brought into above forms by suitable reductions or transformations.

7. If $\int \frac{x^4-2}{x^2\sqrt{x^4+x^2+2}}\,dx = \sqrt{ax^2+\frac{b}{x^2}+1}+c$, then $a+b=$ ______.

8. If $\int \frac{(x-1)}{(x+1)\sqrt{x^3+x^2+x}}\,dx$

$=2\tan^{-1}\left(x+\frac{1}{x}+1\right)^{1/\lambda}+c$, then $\lambda=$ ________.

Question Stem for Question Nos. 9 and 10

If $ABCD$ be a cyclic quadrilateral then $A+C=\pi/2$ and $B+D=\frac{\pi}{2}$. Also, $[\vec{a}\,\vec{b}\,\vec{c}]=\vec{a}\cdot(\vec{b}\times\vec{c})$ and $\vec{a}\perp\vec{b}\Rightarrow\vec{a}\cdot\vec{b}=0$.

9. If $\vec{u}=(q-r, r-p, p-q)$ and $\vec{v}=\left(\frac{1}{a},\frac{1}{b},\frac{1}{c}\right)$ and a, b, c are T_p, T_q, T_r of an H.P. then the angle between the vectors $\vec{u}$ and $\vec{v}$ is $m°$, where $m=$ ________.

10. If $\vec{u}=(q-r, r-p, p-q)$ and $\vec{v}=(\log a^2, \log b^2, \log c^2)$ and a,b,c are T_p, T_q, T_r of a G.P., then angle between vectors $\vec{u}$ and $\vec{v}$ is $p°$, where $p=$ ________.

SECTION 3

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***

11. If $a>0$, $2\log_x a+\log_{ax} a+3\log_{a^2x} a=0$, then $x=$

(a) $a^{1/2}$ (b) $a^{-1/2}$ (c) $a^{-2/3}$ (d) $a^{-4/3}$

12. If OT and ON are perpendiculars drawn from the origin to the tangent and normal to the curve $x=a\sin^3 t$, $y=a\cos^3 t$ an arbitrary point, then

(a) $4OT^2+ON^2=a^2$

(b) the length of the tangent $=\left|\frac{y}{\cos t}\right|$

(c) the length of the normal $=\left|\frac{y}{\sin t}\right|$

(d) none of these

13. If m & M are the smallest & greatest value of the function $f(x)=\frac{\sin x}{x}$ defined in $\left[\frac{\pi}{4},\frac{\pi}{3}\right]$, then

(a) $m=f\left(\frac{\pi}{3}\right)$

(b) $M=f\left(\frac{\pi}{4}\right)$

(c) $\frac{\sqrt{3}}{8}\le\int_{\pi/4}^{\pi/3} f(x)dx\le\frac{\sqrt{2}}{6}$

(d) none of these

14. If $f(x)=\begin{cases}\frac{x\log\cos x}{\log(1+x^2)}, & \text{for } x\ne 0\\ 0, & \text{for } x=0\end{cases}$, then

(a) $f(x)$ is continuous at $x=0$

(b) $f(x)$ is continuous but not differentiable at $x=0$

(c) $f(x)$ is differentiable at $x=0$

(d) none of these

15. If $x\cos(y/x)(y\,dx+x\,dy)=y\sin(y/x)(x\,dy-y\,dx)$, $y(1)=2\pi$, then the value of $4\frac{y(4)}{\pi}\cos\left(\frac{y(4)}{4}\right)$ is

(a) -2 (b) 0 (c) 1 (d) 2

16. If $f(\alpha)=\lim\limits_{x\to 2}(\sin^x\alpha+\cos^x\alpha)^{\frac{1}{(x-2)}}$ for $\alpha\in\left[0,\frac{\pi}{2}\right]$, then

(a) $f(0)=1$ (b) $f\left(\frac{\pi}{2}\right)=1$

(c) $f(\alpha)=(\cos\alpha)^{\cos^2\alpha}\cdot(\sin\alpha)^{\sin^2\alpha}$ if $\alpha\in\left(0,\frac{\pi}{2}\right)$

(d) $f(\alpha)=\frac{(\sin\alpha)^{\sin^2\alpha}}{(\cos\alpha)^{\cos^2\alpha}}$ if $\alpha\in\left(0,\frac{\pi}{2}\right)$

SECTION 4

- ***This section contains THREE (03) questions.***
- ***The answer to each question is a NON-NEGATIVE INTEGER.***

17. The maximum value of the expression $\frac{1}{\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta}$ is ______.

18. If $f(x)=\begin{cases}\frac{(5^x-1)^2}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x}{3}\right)}, & x\ne 0\\ 9(\log_e 5)^2, & x=0\end{cases}$

is continuous at $x=0$, then value of 'a' equals __________.

19. Let $A(1, 2, -3)$ and $B(3, -4, 2)$ then the moment about the point $C(-2, 4, -6)$ of the force represented in magnitude and position by $\overrightarrow{AB}$ is $\frac{\sqrt{341}}{\lambda}$. Then value of λ is ______.

PAPER-2

SECTION 1

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***

1. $f: R \to R$ and $g: R \to R$ are one to one, real valued functions. Let $\phi(x) = [f(x) + f(-x)][g(x) - g(-x)]$, then

(a) $\phi(x)$ is an even function

(b) $\phi(x)$ is an odd function

(c) $\phi'(x)$ is an odd function

(d) $\phi'(x)$ is an even function.

2. A function $f : R \to R$ is defined by $f(x) = \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$. Then the integral value of α for which f is onto.

(a) 3 (b) 4 (c) 1 (d) 10

3. Let $\vec{A}$ be vector parallel to the line of intersection of planes P_1 and P_2 through the origin. P_1 is parallel to the vectors $\vec{a} = 2\hat{j} + 3\hat{k}$ and $\vec{b} = 4\hat{j} - 3\hat{k}$ and P_2 is parallel to the vectors $\vec{c} = \hat{j} - \hat{k}$ and $\vec{d} = 3\hat{i} + 3\hat{j}$. The angle between $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is

(a) $\pi/2$ (b) $\pi/4$ (c) $\pi/6$ (d) $3\pi/4$

4. Find the area enclosed by the curves $y = 2 - |2 - x|$ and $y = 3/|x|$

(a) $3 - 4\log 3$ (b) $4 - 3\log 3/2$

(c) $2 - \log 3^{3/2}$ (d) $2 - \log 3$

5. If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then

(a) $f(x)$ is continuous everywhere in the domain

(b) $f(x)$ is differentiable at $x = 0$

(c) $f(x)$ is not differentiable at $x = 0$

(d) none of these

6. If $\lim\limits_{x \to 0} \dfrac{ae^x + b\cos x + ce^{-x}}{e^{2x} - 2e^x + 1} = 4$, then

(a) $a = 2$ (b) $b = -4$

(c) $c = 2$ (d) $a + b + c = -8$

SECTION 2

- ***This section contains THREE (03) question stems.***
- ***There are TWO (02) questions corresponding to each question stem.***
- ***The answer to each question is a NUMERICAL VALUE.***

Question Stem for Question Nos. 7 and 8

Let $f(x) = \begin{cases} 2x + a : x \geq -1 \\ bx^2 + 3 : x < -1 \end{cases}$ and

$g(x) = \begin{cases} x + 4 \quad : 0 \leq x \leq 4 \\ -3x - 2 : -2 < x < 0 \end{cases}$ functions

7. The value of $a \times b$ is greater than ________ if $g(f(x))$ is not defined.

8. If domain of $g(f(x))$ is $[-1, -4]$, then the value of a is ______.

Question Stem for Question Nos. 9 and 10

Integrals of the form

$\displaystyle\int \frac{x^2 + 1}{x^4 + Kx^2 + 1}\,dx, \int \frac{x^2 - 1}{x^4 + Kx^2 + 1}\,dx$, where $K \in R$ can be integrated by the following method.

(i) Divide the numerator and denominator by x^2.

(ii) Put $x + \dfrac{1}{x}$ or $x - \dfrac{1}{x} = z$ as required.

9. $\displaystyle\int \frac{x^2 - 1}{x^4 + x^2 + 1}\,dx = \frac{1}{k}\log\left|\frac{x^2 - x + 1}{x^2 + x + 1}\right| + c$, then $k =$ ____.

10. $\displaystyle\int \frac{dx}{x^4 + 1}$ is equal to $= \dfrac{1}{a}\tan^{-1}\dfrac{x^2 - 1}{\sqrt{2}x}$

$-\dfrac{1}{b}\log\left|\dfrac{x^2 - \sqrt{2x} + 1}{x^2 + \sqrt{2x} + 1}\right| + c$, then $a \times b =$ _____.

Question Stem for Question Nos. 11 and 12

Let $a\hat{i} + \hat{j} + \hat{k},\ \hat{i} + b\hat{j} + \hat{k},\ \hat{i} + \hat{j} + c\hat{k}$ be coplanar vectors where $(a - 1)(b - 1)(c - 1) \neq 0$.

11. The value of $\dfrac{1}{a-1} + \dfrac{1}{b-1} + \dfrac{1}{c-1} =$ ______.

12. The value of $\dfrac{a+1}{a-1} + \dfrac{b+1}{b-1} + \dfrac{c+1}{c-1} =$ ______.

SECTION 3

- ***This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO(02) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.***
- ***For each question, choose the option corresponding to the correct answer.***

Paragraph for Q. 13 and 14

If a continuous function f, defined on the real line R, assumes positive and negative values in R, then the equation $f(x) = 0$ has a root in R. For example, if it is known that a continuous function f on R is positive at some point and its minimum value is negative, then the equation $f(x) = 0$ has a root in R.

Consider $f(x) = ke^x - x$ for all real x, where k is a real constant.

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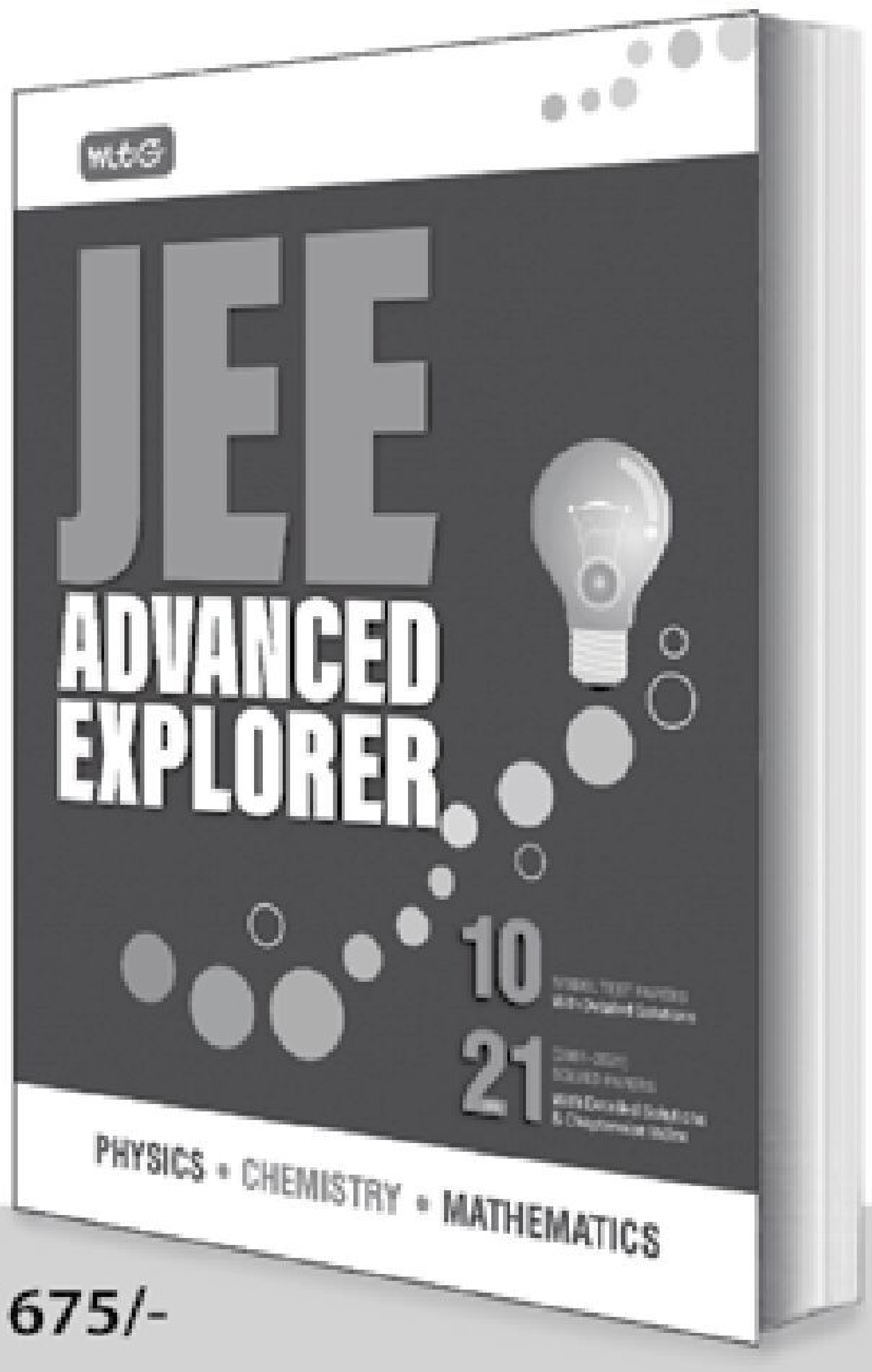
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13. The line $y = x$ meets $y = ke^x$ for $k \le 0$ at

(a) no point (b) one point

(c) two points (d) more than two points

14. The positive value of k for which $ke^x - x = 0$ has only one root is

(a) $\frac{1}{e}$ (b) 1 (c) e (d) $\log_e 2$

Paragraph for Q. 15 and 16

Consider the function $f(x)$ such that

$$f(x) = x^2 + \int_0^1 (x+t)f(t)dt$$

15. The area bounded by the x-axis, the tangent and normal at $x = 0$ on the curve $y = f(x)$ is

(a) $\frac{3757}{60}$ (b) $\frac{3757}{90}$ (c) $\frac{3757}{180}$ (d) $\frac{3757}{120}$

16. The area bounded by the curve $y = f(x)$ and the x-axis is

(a) $\frac{341}{3}\sqrt{109}$ (b) $\frac{109}{18}\sqrt{\frac{109}{3}}$

(c) $341\sqrt{\frac{109}{3}}$ (d) $\frac{109}{9}\sqrt{\frac{109}{3}}$

SECTION 4

- ***This section contains THREE (03) questions.***
- ***The answer to each question is a NON-NEGATIVE INTEGER.***

17. If $y^{y^{y^{\cdot^{\cdot^{\cdot^{\infty}}}}}} = \log_e(x + \log_e(x + ...))$, and $\frac{dy}{dx} = \frac{1-\log a}{k\sqrt{2}(e^b - 1)}$ at ($x = e^2 - 2$, $y = \sqrt{2}$) then the value of $(ab)^k$ is equal to

18. $f(x) = \min\{\{x\}, \{-x\}\}$ where $\{\cdot\}$ denotes the fractional part function, then the area bounded by $f(x)$, axis & $x = -4034$, $x = 4034$ is ________.

19. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c})$. If magnitudes of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are $\sqrt{2}, 1$ and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta\left(0 < \theta < \frac{\pi}{2}\right)$, then the value of 1 + tanθ is equal to

SOLUTIONS

PAPER-1

1. (b) : We have, $\lim_{h\to 0} 2\left\{\frac{\frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{6}+h\right) - \frac{1}{2}\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\frac{\sqrt{3}}{2}\cos h - \frac{1}{2}\sin h\right)}\right\}$

$$= \lim_{h\to 0} 2\left\{\frac{\cos\frac{\pi}{6}\sin\left(\frac{\pi}{6}+h\right) - \sin\frac{\pi}{6}\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\cos\frac{\pi}{6}\cos h - \sin\frac{\pi}{6}\sin h\right)}\right\}$$

$$= \lim_{h\to 0} 2\left\{\frac{\sin\left(\frac{\pi}{6}+h-\frac{\pi}{6}\right)}{\sqrt{3}h\cos\left(\frac{\pi}{6}+h\right)}\right\} = \lim_{h\to 0}\frac{2}{\sqrt{3}}\left(\frac{\sin h}{h}\right)\frac{1}{\cos\left(\frac{\pi}{6}+h\right)}$$

$$= \frac{2}{\sqrt{3}\cos\frac{\pi}{6}} = \frac{2}{\sqrt{3}\cdot\frac{\sqrt{3}}{2}} = \frac{4}{3}$$

2. (b) : Let $\cot^{-1}\sqrt{\frac{1-x}{x}} = \theta$; $\cot\theta = \sqrt{\frac{1-x}{x}}$

$$\therefore \sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right) = \sin\theta$$

Since, $\sin\theta = \frac{1}{\sqrt{1+\cot^2\theta}} = \frac{1}{\sqrt{1+\frac{1-x}{x}}} = \sqrt{x}$

Now, $\tan^{-1}\left[\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right] = \tan^{-1}\sqrt{x}$

$$\Rightarrow 2\tan^{-1}\left[\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right] = 2\tan^{-1}\sqrt{x}$$

$$= \tan^{-1}\left(\frac{2\sqrt{x}}{1-x}\right) = \phi \text{ (say)} \Rightarrow \tan\phi = \frac{2\sqrt{x}}{1-x}$$

$$\therefore \cos\left(2\tan^{-1}\left(\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)\right) = \cos\phi$$

and $\cos\phi = \frac{1}{\sqrt{1+\tan^2\phi}}$

$$=\frac{1}{\sqrt{1+\frac{4x}{(1-x)^2}}}=\frac{(1-x)}{(1+x)}=f(x)\ \text{(say)}$$

$$\Rightarrow\quad f'(x)=\frac{-1\times(1+x)-(1-x)}{(1+x)^2}=\frac{-2}{(1+x)^2}$$

$$\Rightarrow\quad (1-x)^2\ f'(x)=\frac{-2(1-x)^2}{(1+x)^2}=-2[f(x)]^2$$

$$\Rightarrow\quad (1-x)^2 f'(x)+2[f(x)]^2=0$$

3. (b) : Let $I=\int\frac{[\sin\theta\cdot\sin 2\theta(\sin^6\theta+\sin^4\theta+\sin^2\theta)\sqrt{2\sin^4\theta+3\sin^2\theta+6}]}{1-\cos 2\theta}d\theta$

$$=\int\frac{[\sin\theta\cdot 2\sin\theta\cos\theta\cdot\sin^2\theta(\sin^4\theta+\sin^2\theta+1)(2\sin^4\theta+3\sin^2\theta+6)^{1/2}]}{2\sin^2\theta}d\theta$$

$$=\int\sin^2\theta\cdot\cos\theta(\sin^4\theta+\sin^2\theta+1)(2\sin^4\theta+3\sin^2\theta+6)^{1/2}d\theta$$

Putting $\sin\theta = t \ \Rightarrow\ \cos\theta\, d\theta = dt$, we get

$$I=\int t^2(t^4+t^2+1)(2t^4+3t^2+6)^{1/2}\,dt$$

$$=\int(t^5+t^3+t)\,t\,(2t^4+3t^2+6)^{1/2}\,dt$$

$$=\int(t^5+t^3+t)\,(2t^6+3t^4+6t^2)^{1/2}\,dt$$

Again put $2t^6 + 3t^4 + 6t^2 = u^2$

$\Rightarrow\quad 12(t^5 + t^3 + t)dt = 2udu$

$$\therefore\quad I=\int(u^2)^{1/2}\frac{2udu}{12}=\int\frac{u^2}{6}du=\frac{u^3}{18}+c$$

$$=\frac{(2t^6+3t^4+6t^2)^{3/2}}{18}+c$$

$$=\frac{1}{18}(2\sin^6\theta+3\sin^4\theta+6\sin^2\theta)^{3/2}+c$$

$$=\frac{1}{18}[2(1-\cos^2\theta)^3+3(1-\cos^2\theta)^2+6(1-\cos^2\theta)]^{3/2}+c$$

$$=\frac{1}{18}\{(1-\cos^2\theta)[2(1-\cos^2\theta)^2+3(1-\cos^2\theta)+6]\}^{3/2}+c$$

$$=\frac{1}{18}[(1-\cos^2\theta)(2\cos^4\theta-7\cos^2\theta+11)]^{3/2}+c$$

$$=\frac{1}{18}[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta]^{3/2}+c$$

4. (c) : (a) $[\vec{a}\,\vec{b}\,\vec{c}]+[\vec{c}\,\vec{a}\,\vec{b}]=2[\vec{a}\,\vec{b}\,\vec{c}]=2(\vec{a}\cdot(\vec{b}\times\vec{c}))$

$=2\vec{a}\cdot\vec{a}=2|\vec{a}|^2=8 \qquad (\because \vec{a}=\vec{b}\times\vec{c})$

(b) Projection of $\vec{a}$ on $\vec{b}\times\vec{c}=\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|}=\frac{\vec{a}\cdot\vec{a}}{|\vec{a}|}=|\vec{a}|=2$

(c) If $\vec{a},\vec{b},\vec{c}$ are mutually perpendicular vectors, then

$|\vec{a}\times\vec{b}|=|\vec{c}|\Rightarrow|\vec{a}||\vec{b}|=|\vec{c}|\Rightarrow|\vec{b}|=\frac{|\vec{c}|}{2}$

Also, $|\vec{b}\times\vec{c}| = |\vec{a}|$

$\Rightarrow\quad |\vec{b}||\vec{c}| = 2 \Rightarrow\ |\vec{c}| = 2$ and $|\vec{b}| = 1$

$|3\vec{a}+\vec{b}-2\vec{c}|^2=(3\vec{a}+\vec{b}-2\vec{c})\cdot(3\vec{a}+\vec{b}-2\vec{c})$

$= 9|\vec{a}|^2 + |\vec{b}|^2 + 4|\vec{c}|^2 = (9 \times 4) + 1 + (4 \times 4)$

$= 36 + 1 + 16 = 53$

(d) $\vec{a}\times((\vec{b}+\vec{c})\times(\vec{b}-\vec{c}))$

$=\vec{a}\times((-\vec{b}\times\vec{c})+(\vec{c}\times\vec{b}))=-2(\vec{a}\times(\vec{b}\times\vec{c}))=-2(\vec{a}\times\vec{a})=\vec{0}$

5. (0) : $\lim_{x\to 1}\frac{f(x)}{(x-1)^2} = 1$

$\Rightarrow\quad f(1) = 0, f'(1) = 0, f''(1) = 2$

$\therefore\quad f(x) = A(x-1)^4 + B(x-1)^3 + (x-1)^2$

$f'(x) = 4A\,(x-1)^3 + 3B(x-1)^2 + 2(x-1)$

$f'(0) = -6 \Rightarrow -4A + 3B - 2 = -6$ ...(i)

$f'(2) = 6 \Rightarrow 4A + 3B + 2 = 6$ ...(ii)

(i), (ii) $\Rightarrow A = 1, B = 0$

$\therefore\quad f(x) = (x-1)^4 + (x-1)^2 \geq 0$

$f(1) = 0$ is the minimum.

6. (3) : $f(x) = (x-1)^4 + (x-1)^2, f'(0) = -6, f(0) = 2$

$f'(x) = 4(x-1)^3 + 2(x-1)$

Subtangent at $x = 0$ is $\left|\frac{f(0)}{f'(0)}\right|=\frac{2}{6}=\frac{1}{3}$

7. (3) : $\int\frac{x-2/x^3}{\sqrt{x^2+1+\frac{2}{x^2}}}dx,$ Put $x^2+\frac{2}{x^2}+1=t$

8. (2) : $\int\frac{x^2-1}{(x+1)^2\sqrt{x^3+x^2+x}}dx=\int\frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}+2\right)\sqrt{x+\frac{1}{x}+1}}dx$

Put $x+\frac{1}{x}+1=t^2$

9. (90) : $\frac{1}{a} = T_p$ of A.P. $= A + (p-1)\,D = (A-D) + pD.$

If θ be the angle between $\vec{u}$ and $\vec{v}$, then

$\vec{u}\cdot\vec{v}=|\vec{u}||\vec{v}|\ \cos\theta = \Sigma\,(q-r)\frac{1}{a}$

$= \Sigma(q-r)\ [(A-D) + pD]$

$= (A-D)\ \Sigma\,(q-r) + D\Sigma\, p\,(q-r) = 0$

$\therefore\quad \cos\theta = 0 \quad$ or $\quad \theta = 90°$

10. (90) : $a = AR^{p-1}$

$\therefore\quad \log a = \log A + (p-1)\log R$

If θ be the angle between $\vec{u}$ and $\vec{v}$, then

$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta = \Sigma(q-r)\log a^2 = 2\Sigma(q-r)\log a$
$= 2\Sigma(q-r)\,[\log A + (p-1)\log R]$
$= 2[\log A\Sigma(q-r) + \log R\Sigma(p-1)(q-r)] = 0$
$\therefore\ \cos\theta = 0$ or $\theta = 90°$

11. (b,d) : $\dfrac{2}{\log_a x} + \dfrac{1}{\log_a ax} + \dfrac{3}{\log_a a^2 x} = 0$

$\Rightarrow\ \dfrac{2}{t} + \dfrac{1}{1+t} + \dfrac{3}{2+t} = 0$, where $t = \log_a x$

$\Rightarrow\ 2(1+t)(2+t) + t(2+t) + 3t(1+t) = 0$

$\Rightarrow\ 6t^2 + 11t + 4 = 0 \Rightarrow t = -\dfrac{1}{2}, -\dfrac{4}{3}$

$\Rightarrow\ x = a^{-1/2}, a^{-4/3}$.

12. (a,b,c) : At any point 't' on the given curve, we have

$$\frac{dy}{dx} = \frac{3a\cos^2 t(-\sin t)}{3a\sin^2 t(\cos t)} = -\cot t$$

The equations of the tangent and normal at 't' are
$x\cos t + y\sin t - (a/2)\sin 2t = 0$
and $x\sin t - y\cos t - a\cos 2t = 0$

$\therefore\ OT = \dfrac{|(a/2)\sin 2t|}{\sqrt{\cos^2 t + \sin^2 t}} = (a/2)\sin 2t$

$\Rightarrow\ 2\cdot OT = a\sin 2t$

and $ON = \dfrac{|a\cos 2t|}{\sqrt{\sin^2 t + \cos^2 t}} = a\cos 2t$

Hence, $4OT^2 + ON^2 = a^2\sin^2 2t + a^2\cos^2 2t = a^2$

Length of the tangent at 't' $= \left|\dfrac{y\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}}{dy/dx}\right|$

$= \left|\dfrac{y\ \mathrm{cosec}\ t}{-\cot t}\right| = \left|\dfrac{y}{\cos t}\right|$

Length of the normal at 't' $= \left|y\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\right|$

$= \left|y\sqrt{1+\cot^2 t}\right| = \left|\dfrac{y}{\sin t}\right|$

13. (a, b, c) : Property : If m & M be the smallest & greatest values of $f(x)$ defined in $[a, b]$, then

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a) \qquad \ldots(*)$$

Now, $f(x) = \dfrac{\sin x}{x}$

$\therefore f'(x) = \dfrac{x\cos x - \sin x}{x^2} = \dfrac{(x-\tan x)\cos x}{x^2} < 0$, for the function defined is $\left[\dfrac{\pi}{4}, \dfrac{\pi}{3}\right]$

$\Rightarrow\ f(x)$ decreases on the interval

$\therefore\ m = f\left(\dfrac{\pi}{3}\right) = \dfrac{3\sqrt{3}}{2\pi}$ and $M = f\left(\dfrac{\pi}{4}\right) = \dfrac{2\sqrt{2}}{\pi}$

Again by (*), we have

$$\frac{3\sqrt{3}}{2\pi}\left(\frac{\pi}{3}-\frac{\pi}{4}\right) \le \int_{\pi/4}^{\pi/3} f(x)dx \le \frac{2\sqrt{2}}{\pi}\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$$

or $\dfrac{3\sqrt{3}}{2\pi}\left(\dfrac{\pi}{12}\right) \le \displaystyle\int_{\pi/4}^{\pi/3} f(x)dx \le \dfrac{2\sqrt{2}}{\pi}\left(\dfrac{\pi}{12}\right)$

or $\dfrac{\sqrt{3}}{8} \le \displaystyle\int_{\pi/4}^{\pi/3} f(x)dx \le \dfrac{\sqrt{2}}{6}$

14. (a, c) : $f(x) = \begin{cases} \dfrac{x\log\cos x}{\log(1+x^2)}, & x \ne 0 \\ 0, & x = 0 \end{cases}$

$$\text{L.H.L.} = \lim_{x\to 0} f(x) = \lim_{h\to 0}\frac{-h\log\cos h}{\log(1+h^2)}$$

$$= -\lim_{h\to 0}\frac{\log\cos h - \dfrac{h}{\cos h}\sin h}{\dfrac{1}{1+h^2}\times 2h}$$

$$= -\lim_{h\to 0}\frac{(\log\cos h - h\tan h)(1+h^2)}{2h}$$

$$= -\lim_{h\to 0}\frac{(-\tan h - \tan h - h\sec^2 h)(1+h^2) + 2h(\log\cos h - h\tan h)}{2}$$

$$= -\frac{1}{2}\big(-0-0-0\big)\big(1+0\big) + 0 = 0$$

$$\text{R.H.L.} = \lim_{h\to 0}\frac{\log\cos h}{\dfrac{\log(1+h^2)}{h^2}\times h} = \lim_{h\to 0}\frac{\log\cos h}{h} = -\lim_{h\to 0}\frac{\tan h}{1} = 0$$

$\therefore\ \lim\limits_{x\to 0} f(x) = 0\ \Rightarrow f$ is continuous at $x = 0$

$$\text{L.H.D.} = \lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$$

$$= \lim_{h\to 0}\frac{\dfrac{-h\log\cos h}{\log(1+h^2)} - 0}{-h} = \lim_{h\to 0}\frac{\log\cos h}{\log(1+h^2)}$$

$$= \lim_{h\to 0}\frac{-\tan h(1+h^2)}{2h} = -\frac{1}{2}$$

$$\text{R.H.D.} = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{\dfrac{h\log\cos h}{\log(1+h^2)}}{h}$$

$$= \lim_{h\to 0}\frac{-\tan h(1+h^2)}{2h} = -\frac{1}{2}$$

$\therefore\ f(x)$ is differentiable at $x = 0$.

15. (d) : The given equation can be written as

$$\frac{dy}{dx}=\frac{y(y\sin(y/x)+x\cos(y/x))}{x(y\sin(y/x)-x\cos(y/x))}$$

Putting $y = vx$ in the above equation, we get

$$v+x\frac{dv}{dx}=\frac{v(v\sin v+\cos v)}{v\sin v-\cos v}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{v(v\sin v+\cos v)}{v\sin v-\cos v}-v=\frac{2v\cos v}{v\sin v-\cos v}$$

$$\Rightarrow \frac{v\sin v-\cos v}{v\cos v}dv=2\frac{dx}{x} \Rightarrow \left(\tan v-\frac{1}{v}\right)dv=2\frac{dx}{x}$$

$$\Rightarrow \log \sec v - \log v = \log x^2 + c$$

$$\Rightarrow \frac{\sec v}{vx^2}=c \Rightarrow \frac{1}{xy}\sec\frac{y}{x}=c$$

$\Rightarrow xy\cos(y/x) = c$ so $y(1) = 2\pi \Rightarrow c = 2\pi$

Hence, $xy\cos(y/x) = 2\pi$

Now, Putting $x = 4$, we have $4\frac{y(4)}{\pi}\cos(y(4)/4)=2$

16. (a, b, c) : $f(\alpha)=\lim\limits_{x\to 2}(\sin^x\alpha+\cos^x\alpha)^{\frac{1}{(x-2)}}$ (1^∞ form)

$$=\begin{cases} e^{\lim\limits_{x\to 2}\frac{\sin^x\alpha+\cos^x\alpha-1}{x-2}}, & \alpha\in\left(0,\frac{\pi}{2}\right)\\ 1, & \alpha=0,\frac{\pi}{2}\end{cases}$$

$$=e^{\lim\limits_{x\to 2}\frac{\sin^x\alpha+\cos^x\alpha-(\sin^2\alpha+\cos^2\alpha)}{x-2}}$$

$$=e^{\lim\limits_{x\to 2}\frac{\sin^2\alpha(\sin^{x-2}\alpha-1)+\cos^2\alpha(\cos^{x-2}\alpha-1)}{x-2}}$$

$$=e^{\sin^2\alpha\log_e\sin\alpha+\cos^2\alpha\log_e\cos\alpha} \quad =e^{\log_e(\sin\alpha)^{\sin^2\alpha}+\log_e(\cos\alpha)^{\cos^2\alpha}}$$

$$=e^{\log_e(\sin\alpha)^{\sin^2\alpha}(\cos\alpha)^{\cos^2\alpha}} \quad =(\sin\alpha)^{\sin^2\alpha}(\cos\alpha)^{\cos^2\alpha}$$

$$\therefore\ f(x)=\begin{cases}(\cos\alpha)^{\cos^2\alpha}\cdot(\sin\alpha)^{\sin^2\alpha}, & \alpha\in\left(0,\frac{\pi}{2}\right)\\ 1, & \alpha=0,\frac{\pi}{2}\end{cases}$$

17. (2) : $\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta$

$$=\frac{1}{2}\big(1-\cos 2\theta+3\sin 2\theta+5(1+\cos 2\theta)\big)$$

$$=\frac{1}{2}\big(6+4\cos 2\theta+3\sin 2\theta\big) \qquad \text{...(i)}$$

The minimum value of $a\cos x + b\sin x + c$ is $c-\sqrt{a^2+b^2}$

$\therefore$ The minimum value of (i) is $\frac{1}{2}\left(6-\sqrt{4^2+3^2}\right)=\frac{1}{2}$

$\therefore$ The maximum value of its reciprocal is 2.

18. (3) : We have, $\lim\limits_{x\to 0}f(x)=\lim\limits_{x\to 0}\frac{(5^x-1)^2}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x}{3}\right)}$

$$=\lim_{x\to 0}\left(\frac{5^x-1}{x}\right)^2\frac{x^2\times a}{\frac{x\sin\left(\frac{x}{a}\right)}{\left(\frac{x}{a}\right)}}\times\frac{1}{\frac{\log\left(1+\frac{x}{3}\right)}{\frac{x}{3}}\times\frac{x}{3}}$$

$$=\lim_{x\to 0}\left(\frac{5^x-1}{x}\right)^2\frac{1\times a}{\left(\frac{\sin\left(\frac{x}{a}\right)}{\frac{x}{a}}\right)}\cdot\frac{3}{\frac{\log\left(1+\frac{x}{3}\right)}{\frac{x}{3}}}$$

$$=\lim_{x\to 0}\left(\frac{5^x-1}{x}\right)^2\frac{a}{\lim\limits_{x\to 0}\left(\frac{\sin\left(\frac{x}{a}\right)}{\frac{x}{a}}\right)}\cdot\frac{3}{\lim\limits_{x\to 0}\log\left(1+\frac{x}{3}\right)^{\frac{3}{x}}}$$

$$=(\log_e 5)^2\cdot\frac{a}{1}\cdot\frac{3}{\log e}=3a(\log_e 5)^2$$

Also, $f(0) = 9(\log_e 5)^2$

Since, $f(x)$ is continuous at $x = 0$

$\therefore\ \lim\limits_{x\to 0}f(x)=f(0)$

$\Rightarrow 3a(\log_e 5)^2 = 9(\log_e 5)^2 \Rightarrow a = 3$

19. (1) : If 'O' is the origin then $\overrightarrow{OA}=\hat{i}+2\hat{j}-3\hat{k}$, $\overrightarrow{OB}=3\hat{i}-4\hat{j}+2\hat{k}$ and $\overrightarrow{OC}=-2\hat{i}+4\hat{j}-6\hat{k}$

$$\therefore\ \overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(3\hat{i}-4\hat{j}+2\hat{k})-(\hat{i}+2\hat{j}-3\hat{k})$$
$$=2\hat{i}-6\hat{j}+5\hat{k}$$

$$\therefore\ \overrightarrow{CA}=\overrightarrow{OA}-\overrightarrow{OC}=(\hat{i}+2\hat{j}-3\hat{k})-(-2\hat{i}+4\hat{j}-6\hat{k})$$
$$=3\hat{i}-2\hat{j}+3\hat{k}$$

$$\therefore\ \text{Required moment}=\overrightarrow{CA}\times\overrightarrow{AB}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-2&3\\2&-6&5\end{vmatrix}$$

$$=\hat{i}(-10+18)-\hat{j}(15-6)+\hat{k}(-18+4)=8\hat{i}-9\hat{j}-14\hat{k}$$

$$\therefore\ |\overrightarrow{CA}\times\overrightarrow{AB}|=|8\hat{i}-9\hat{j}-14\hat{k}|$$
$$=\sqrt{8^2+(-9)^2+(-14)^2}=\sqrt{341}$$

PAPER-2

1. (b, d) : Given, $\phi(x) = [f(x) + f(-x)]\,[g(x) - g(-x)]$

$\therefore\quad \phi(-x) = [f(-x) + f(x)][g(-x) - g(x)] = -\phi(x)$

$\Rightarrow\quad \phi(x)$ is an odd function $\Rightarrow$ $\phi'(x)$ is an even function.

[$\because$ Differentiation of an odd function is an even function]

2. (a, b, d) : Since, $f : R \to R$ is an onto mapping.

$\therefore\quad$ Range of $f = R$

$\Rightarrow\quad \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$ assumes all real value of x.

Let $y = \dfrac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$

Then, y assumes all real values for real values of x.

$\Rightarrow\quad \alpha y + 6xy - 8x^2 y = \alpha x^2 + 6x - 8\ \forall\ y \in R$

$\Rightarrow\quad x^2(\alpha + 8y) + 6x(1 - y) - (8 + \alpha y) = 0\ \forall\ y \in R$

we know, above equation assumes all real values

$\Rightarrow\quad D \geq 0$

So, $36(1 - y)^2 + 4(\alpha + 8y)(8 + \alpha y) \geq 0$

$\Rightarrow\quad 4[9(1 - 2y + y^2) + (8\alpha + \alpha^2 y + 64y + 8\alpha y^2)] \geq 0$

$\Rightarrow\quad [9 - 18y + 9y^2 + 8\alpha + \alpha^2 y + 64y + 8\alpha y^2] \geq 0$

$\Rightarrow\quad [y^2(8\alpha + 9) + y(\alpha^2 + 46) + (8\alpha + 9)] \geq 0$

we know, if $ax^2 + bx + c > 0\ \forall\ x$ then $a > 0$ and $D < 0$

So, $(\alpha^2 + 46)^2 - 4(8\alpha + 9)(8\alpha + 9) \leq 0$ and $(8\alpha + 9) > 0$

$\Rightarrow\quad (\alpha^2 + 46)^2 - [2(8\alpha + 9)]^2 \leq 0$ and $\alpha > -9/8$

$\Rightarrow\quad (\alpha^2 + 46 - 16\alpha - 18)(\alpha^2 + 46 + 16\alpha + 18) \leq 0$ and $\alpha > -9/8$

$\quad (\alpha^2 - 16\alpha + 28)(\alpha^2 + 16\alpha + 64) \leq 0$ and $\alpha > -9/8$

$\Rightarrow\quad (\alpha - 14)(\alpha - 2)(\alpha + 8)^2 \leq 0$ and $\alpha > -9/8$

$\quad \alpha \in [2, 14] \cup \{-8\}$ and $\alpha > -9/8$

Thus, $\alpha \in [2, 14]$

Hence, f is onto when $\alpha \in [2, 14]$.

3. (b, d) : Plane P_1 is parallel to $\vec{a}$ and $\vec{b}$. The normal to P_1 is along $\vec{a} \times \vec{b}$. Plane P_2 is parallel to $\vec{c}$ and $\vec{d}$. The normal to P_2 is along $\vec{c} \times \vec{d}$.

$\vec{A}$ is along the line of intersection of planes P_1 and P_2

$\therefore\quad \vec{A}$ is along $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d})$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 3 \\ 0 & 4 & -3 \end{vmatrix} = -18\hat{i} \text{ and}$$

$$\vec{c} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ 3 & 3 & 0 \end{vmatrix} = 3\hat{i} - 3\hat{j} - 3\hat{k} = 3(\hat{i} - \hat{j} - \hat{k})$$

$\therefore\quad \vec{A}$ is along $\hat{i} \times (\hat{i} - \hat{j} - \hat{k}) = \hat{j} - \hat{k}$

Let the angle between $\vec{A}$ and $2\hat{i} + \hat{j} - 2\hat{k}$ is θ, then

$$\cos\theta = \frac{\vec{A}}{|\vec{A}|} \cdot \frac{(2\hat{i} + \hat{j} - 2\hat{k})}{3}$$

$$= \pm\frac{(j - \hat{k}) \cdot (2\hat{i} + \hat{j} - 2\hat{k})}{3\sqrt{2}} = \frac{\pm 3}{3\sqrt{2}} = \pm\frac{1}{\sqrt{2}}$$

and $\cos\theta = \pm\dfrac{1}{\sqrt{2}} \Rightarrow \theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}$.

4. (b, c) : Here we have two mod $|x|$ and $|2 - x|$ hence we consider the following cases

$x > 2,\ x < 2,\ x > 0,\ x < 0$

I. $\begin{cases} y = 2 - (2 - x) = x, & x < 2 \\ y = 2 + (2 - x) = 4 - x, & x > 2 \end{cases}$

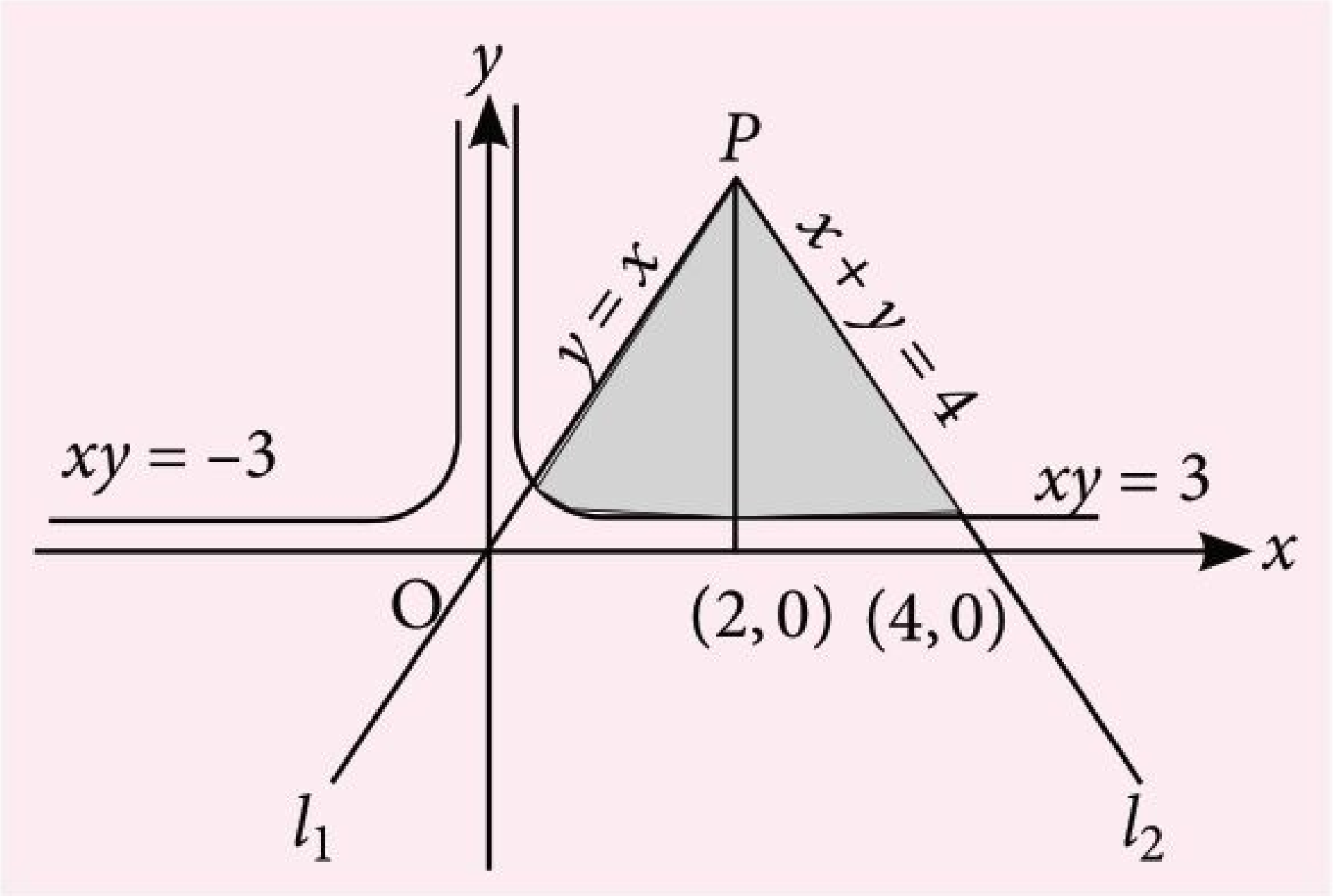
II. $\left\{ y = \dfrac{-3}{x}, x < 0,\ \ y = \dfrac{3}{x}, x > 0 \right.$

The graph consists of two lines

$l_1 : y = x,\ x < 2,\ l_2 : x + y = 4,\ x > 2$

and two hyperbolas

$xy = -3,\ x < 0$ and $xy = 3,\ x > 0$



The hyperbola $xy = 3$ meets the line l_1 *i.e.* $y = x$ at $x^2 = 3\ \ \therefore x = \pm\sqrt{3}$

$\therefore x = \sqrt{3}$ only as $x < 2$ and $x > 0$

It meets the line l_2 *i.e.* $x + y = 4$ where

$x + \dfrac{3}{x} = 4$ or $x^2 - 4x + 3 = 0\ \ \therefore x = 1, 3$

We choose $x = 3$ only as $x > 0$ and $x > 2$

$$\therefore \text{ Required area} = \int_{\sqrt{3}}^{2}\left(x - \frac{3}{x}\right)dx + \int_{2}^{3}\left(4 - x - \frac{3}{x}\right)dx$$

$$= \frac{4 - 3\log 3}{2}$$

5. (a, b) : We have, $f(x) = x(\sqrt{x} - \sqrt{x+1})$

Let us check differentiability of $f(x)$ at $x = 0$

$$Lf'(0) = \lim_{h \to 0}\frac{(0-h)\left[\sqrt{0-h} - \sqrt{0-h+1}\right] - 0}{-h}$$

$$= \lim_{h \to 0}[\sqrt{-h} - \sqrt{-h+1}] = 0 - \sqrt{1} = -1$$

$$Rf'(0)=\lim_{h\to 0}\frac{(0+h)\left[\sqrt{0+h}-\sqrt{0+h+1}\right]-0}{h}$$

$$=\lim_{h\to 0}\sqrt{h}-\sqrt{h+1}=-1$$

Since $Lf'(0) = Rf'(0)$ $\therefore$ f is differentiable at $x = 0$.

6. (a, b, c) : $\displaystyle\lim_{x\to 0}\frac{ae^x+b\cos x+ce^{-x}}{e^{2x}-2e^x+1}=4$

$$\Rightarrow \lim_{x\to 0}\frac{a\left(1+x+\frac{x^2}{2}+...\right)+b\left(1-\frac{x^2}{2}+...\right)+c\left(1-x+\frac{x^2}{2}+...\right)}{x^2}$$

$$\times\left(\frac{x}{e^x-1}\right)^2=4$$

$\Rightarrow a + b + c = 0,\ a - c = 0 \Rightarrow a = c$

$\Rightarrow \dfrac{a-b+c}{2}=4 \ \Rightarrow\ a - b + c = 8 \Rightarrow 2b = -8 \Rightarrow b = -4$

7. (50) : For $g(x)=\begin{cases}x+4 & : \ 0\le x\le 4\\ -3x+2 & : \ -2<x<0\end{cases}$

Maximum value is 8. So, $g(f(x))$ is not defined if (i) $-2 + a > 8$ and (ii) $b + 3 > 8$

$\therefore$ $a > 10$ and $b > 5$

8. (0) : $-1 \le x < 2 \Rightarrow -2 < 2x \le 4$

$\Rightarrow$ $-2 + a \le 2x + a \le 4 + a$

$\Rightarrow$ $-2 + a \le -2$ and $4 + a \le 4$

i.e., $a = 0$ and b can take any value.

9. (2) : $\displaystyle I=\int\frac{x^2-1}{x^4+x^2+1}dx=\int\frac{\left(1-\frac{1}{x^2}\right)}{x^2+1+\frac{1}{x^2}}dx$

Put $\displaystyle x+\frac{1}{x}=t \ \Rightarrow\ I=\int\frac{dt}{t^2-1}$

10. (16) : $\displaystyle\int\frac{dx}{x^4+1}=\frac{1}{2}\left[\int\frac{x^2+1}{x^4+1}dx-\int\frac{x^2-1}{x^4+1}dx\right]$

11. (–1) : $\begin{vmatrix}a & 1 & 1\\ 1 & b & 1\\ 1 & 1 & c\end{vmatrix}=0$

$$\Rightarrow\begin{vmatrix}a & 1 & 1\\ 1-a & b-1 & 0\\ 1-a & 0 & c-1\end{vmatrix}=0\ \begin{bmatrix}R_2\to R_2-R_1\\ R_3\to R_3-R_1\end{bmatrix}$$

Dividing C_1 by $a - 1$, C_2 by $b - 1$, C_3 by $c - 1$, we get

$$\begin{vmatrix}\frac{a}{a-1} & \frac{1}{b-1} & \frac{1}{c-1}\\ -1 & 1 & 0\\ -1 & 0 & 1\end{vmatrix}=0$$

Expanding by R_1, $\dfrac{a}{a-1}+\dfrac{1}{b-1}+\dfrac{1}{c-1}=0$

Subtracting 1 from the first fraction, $\displaystyle\sum\frac{1}{a-1}=-1.$

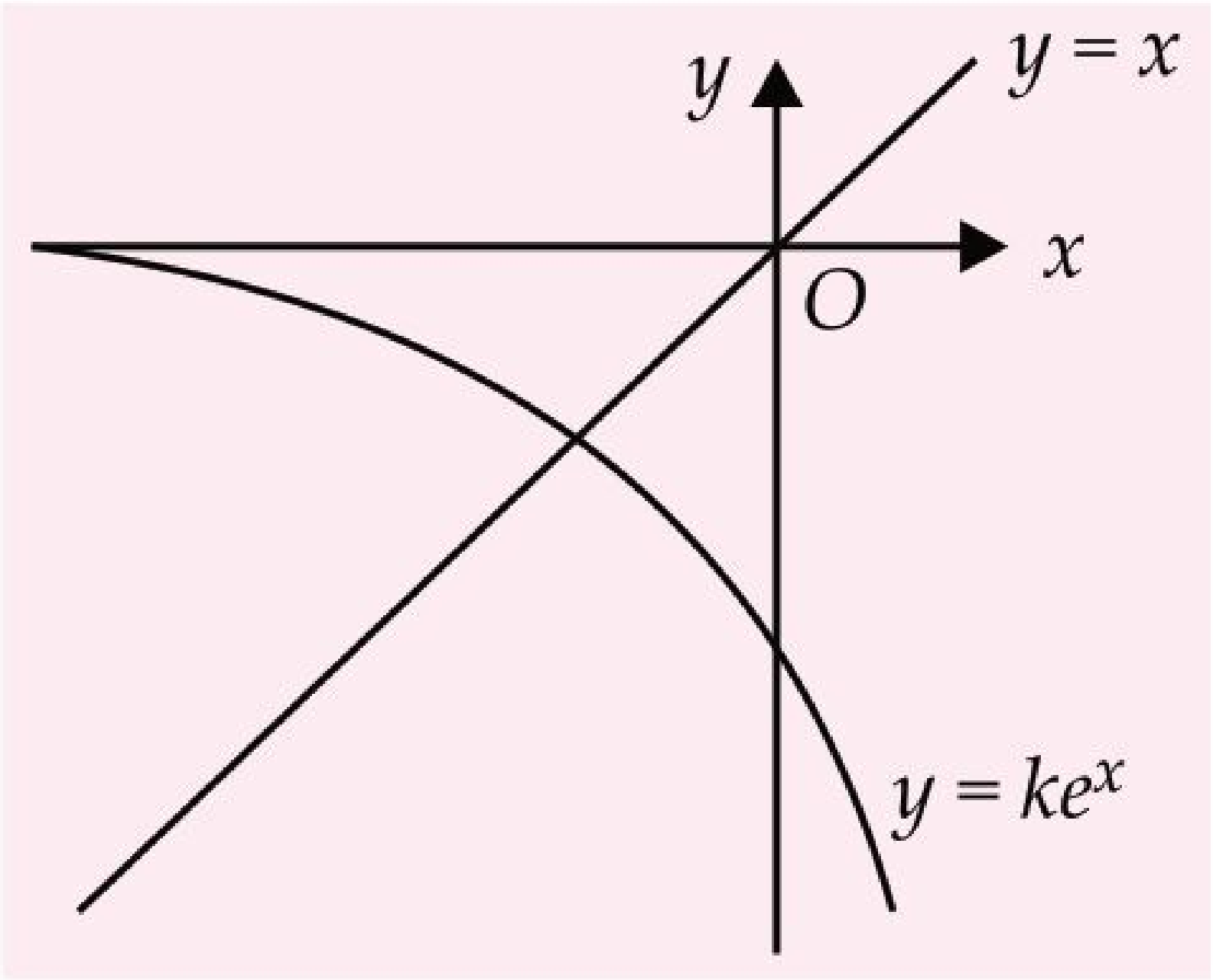
12. (1) : $\displaystyle\sum\frac{a+1}{a-1}=\sum\left(1+\frac{2}{a-1}\right)=\sum 1+2\sum\frac{1}{a-1}$

$= 3 - 2 = 1.$

13. (b) : We have $y = k\cdot e^x$

For $k = 0$, line $y = x$ meets $y = 0$, *i.e.*, the x-axis, only at one point.

For $k < 0$, $y = ke^x$ meets $y = x$ only once as shown in the figure.

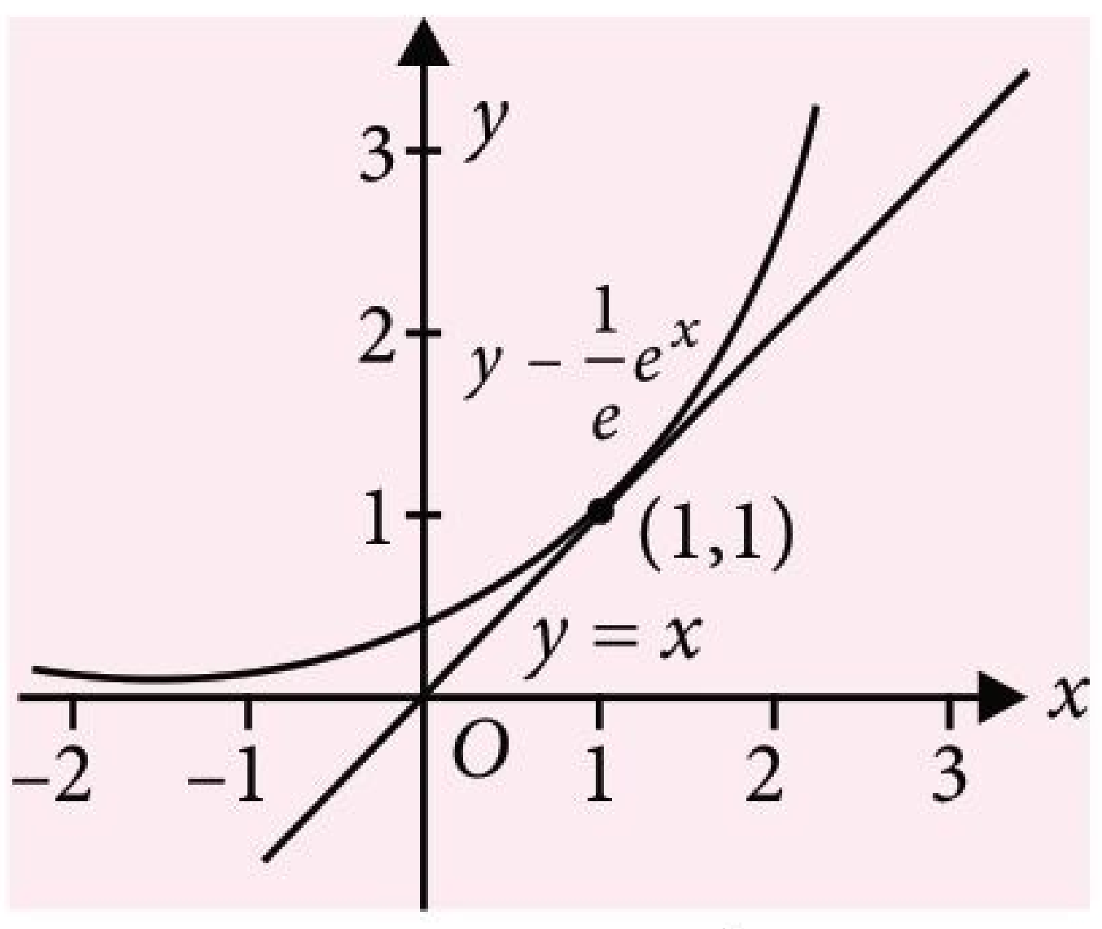


14. (a) : Let $f(x) = ke^x - x$.

Now, for $f(x) = 0$ and to have only one root means the line $y = x$ must be tangential to the curve $y = ke^x$.

Let it be so at (x_1, y_1). Then

$\displaystyle\left(\frac{dy}{dx}\right)_{x_1}=ke^{x_1}=1$ $\quad(\because y = x$ touches curve at $(x_1, y_1))$



$e^{x_1}=\dfrac{1}{k}$

Also, $y_1=ke^{x_1}$

and $y_1 = x_1$

$\therefore$ $x_1 = 1$

$\Rightarrow 1 = ke \Rightarrow k = 1/e$

15. (c) : $y(x) = x^2 + ax + b$, where $a=\int_0^1 y(t)\,dt$, $b=\int_0^1 t\,y(t)\,dt$

$\therefore$ $a=\dfrac{1}{3}+\dfrac{a}{2}+b,\ b=\dfrac{1}{4}+\dfrac{a}{3}+\dfrac{b}{2},$

$\Rightarrow$ $\dfrac{a}{2}-b=\dfrac{1}{3},\ \dfrac{b}{2}-\dfrac{a}{3}=\dfrac{1}{4}$

Solving, $a = -5$, $b=-\dfrac{17}{6}$

$\therefore f(x)=x^2-5x-\dfrac{17}{6}$ $\Rightarrow$ $f'(x) = 2x - 5$

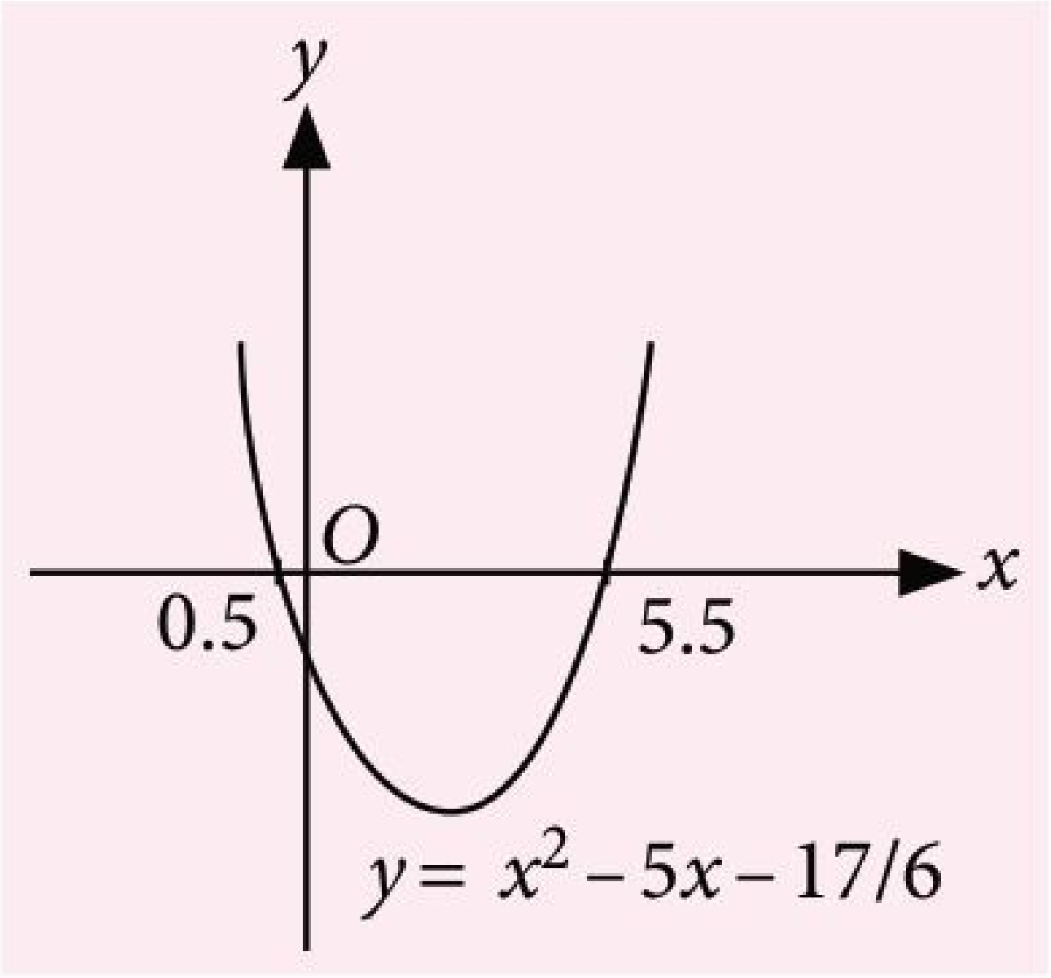
Tangent at $x = 0$ is $y+\dfrac{17}{6}=-5x$ which meets the x-axis at $\left(\dfrac{-17}{30},0\right)$

The normal $x = 0$ is $y+\frac{17}{6} = \frac{x}{5}$ which meets the x-axis at $\left(\frac{85}{6}, 0\right)$

$$\text{Area} = \frac{1}{2}\left(\frac{17}{6}\right)\left(\frac{85}{6}+\frac{17}{30}\right) = \frac{3757}{180}$$

16. (b) : $x^2 - 5x - \frac{17}{6} = 0$ gives the roots

$$\alpha = \frac{5}{2}-\frac{1}{2}\sqrt{\frac{109}{3}},\ \beta = \frac{5}{2}+\frac{1}{2}\sqrt{\frac{109}{3}}$$



Required area,

$$A = -\int_\alpha^\beta f(x)\,dx$$

$$= -\int_\alpha^\beta \left(x^2 - 5x - \frac{17}{6}\right)dx$$

$$= -\left[\frac{x^3}{3} - \frac{5x^2}{2} - \frac{17}{6}x\right]_\alpha^\beta$$

$$= \frac{-(\beta-\alpha)}{6}[2(\alpha^2+\beta^2+\alpha\beta) - 15(\alpha+\beta) - 17]$$

$$= \frac{-(\beta-\alpha)}{6}[2(\alpha+\beta)^2 - 2\alpha\beta - 15(\alpha+\beta) - 17]$$

But $\alpha + \beta = 5$, $\alpha\beta = \frac{-17}{6}$

$$\therefore\ A = \frac{-1}{6}\sqrt{\frac{109}{3}}\left[50+\frac{17}{3}-75-17\right] = \frac{109}{18}\sqrt{\frac{109}{3}}$$

17. (16) : Let $y^{y^{y^{\cdot^{\cdot^{\cdot^{\infty}}}}}} = \log_e(x + \log_e(x + ...)) = v$

$\therefore\ y^v = \log_e(x + v) = v \Rightarrow y = v^{1/v}$ and $x = e^v - v$

$$\Rightarrow\ \frac{dy}{dv} = v^{1/v}\left(\frac{d}{dv}\left(\frac{1}{v}\log v\right)\right)$$

$$= v^{1/v}\left(\frac{1}{v}\cdot\frac{1}{v} - \frac{1}{v^2}\log v\right) = v^{\frac{1}{v}-2}(1 - \log v)$$

and $\frac{dx}{dv} = e^v - 1$ $\therefore$ $\frac{dy}{dx} = \frac{v^{\frac{1}{v}-2}(1-\log v)}{e^v - 1}$

Now, $v^{1/v} = \sqrt{2}$ & $e^v - v = e^2 - 2$

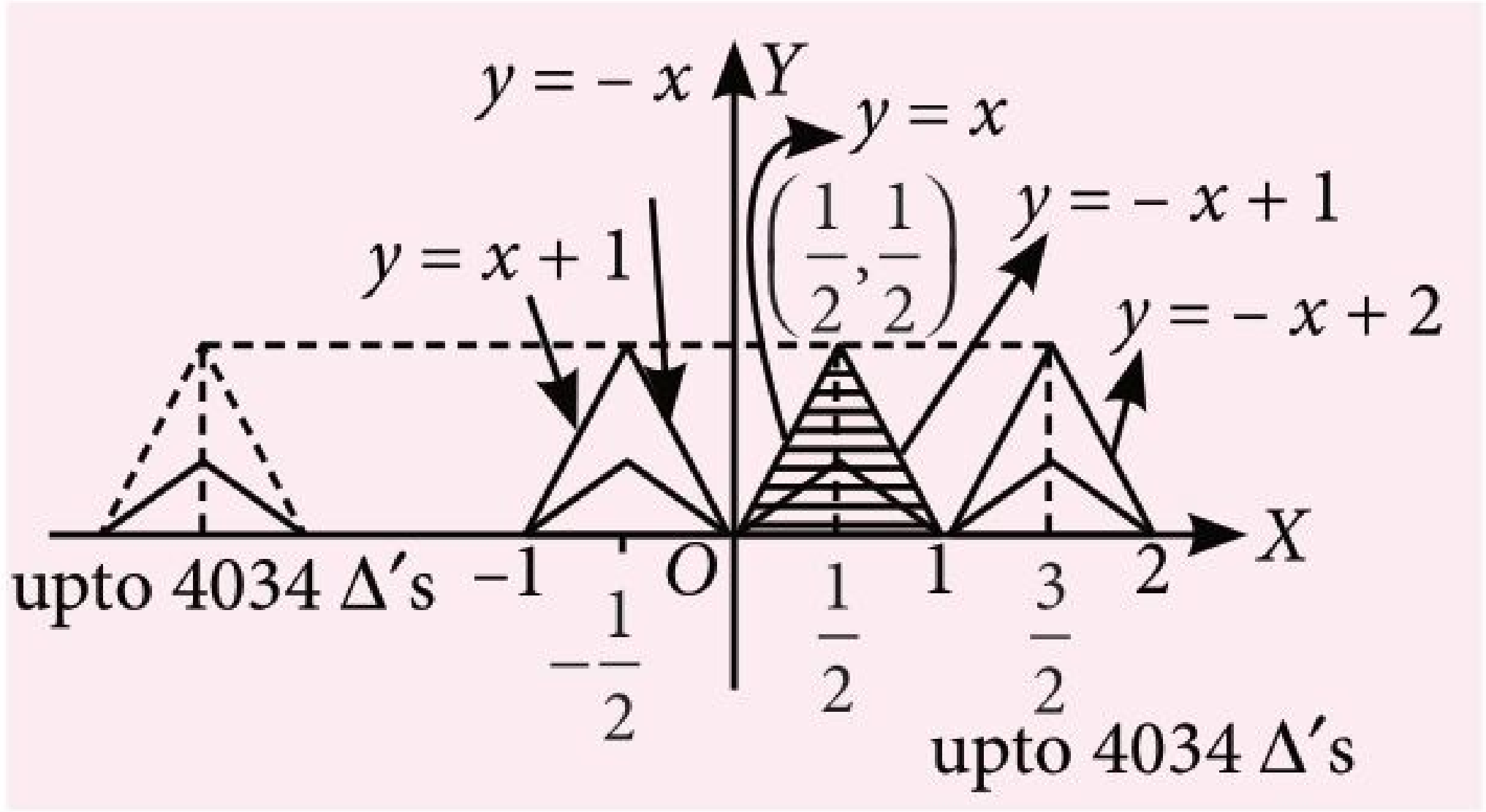
$\Rightarrow\ v = 2$

$$\therefore\ \left(\frac{dy}{dx}\right)_{(e^2-2,\sqrt{2})} = \left(\frac{dy}{dx}\right)_{v=2} = \frac{1-\log 2}{2\sqrt{2}(e^2-1)}$$

$(ab)^k = (2 \times 2)^2 = 16$

18. (2017) : Definition $\{x\} = x - [x]$ is a periodic function with period 1.

$$\text{Now, } \min\{\{x\}, \{-x\}\} = \begin{cases} x, & \forall\ 0 \le x \le \frac{1}{2} \\ -x+1, & \forall\ \frac{1}{2} \le x \le 1 \end{cases}$$



$$\therefore\ \text{Required area} = \int_{-4034}^{4034} \min\{\{x\},\{-x\}\}dx$$

$$= 2\int_0^{4034} \min\{\{x\},\{-x\}\}dx \ \text{(using property)}$$

$$= 2 \times 4034\int_0^1 \min\{\{x\},\{-x\}\}dx \quad \text{(using property)}$$

$$= 2 \times 4034 \times \left[\int_0^{1/2} x\,dx + \int_{1/2}^1 (-x+1)dx\right]$$

$$= 2 \times 4034\left[\frac{1}{2}(x^2)_0^{1/2} + \left(-\frac{1}{2}\right)((-x+1)^2)_{1/2}^1\right]$$

$$= 2 \times 4034\left[\frac{1}{8} + \left(-\frac{1}{2}\right)\left(0 - \frac{1}{4}\right)\right]$$

$$= 2 \times 2 \times 2017\left(\frac{2}{8}\right) = 2017 \text{ sq. units}$$

19. (2) : We have, $\vec{a} = \vec{b} \times (\vec{b} \times \vec{c}) = (\vec{b}\cdot\vec{c})\,\vec{b} - (\vec{b}\cdot\vec{b})\,\vec{c}$

$\Rightarrow\ \vec{a} = (2\cos\theta)\vec{b} - \vec{c}$ ...(i)

[$\because\ \vec{b}\cdot\vec{c} = |\vec{b}||\vec{c}|\cos\theta$, where θ is the angle between $\vec{b}$ and $\vec{c}$]

Now, taking dot product with $\vec{b}$ in (i), we get

$$\vec{a}\cdot\vec{b} = (2\cos\theta)\,\vec{b}\cdot\vec{b} - \vec{c}\cdot\vec{b}$$

$\Rightarrow\ \vec{a}\cdot\vec{b} = 2\cos\theta - 2\cos\theta = 0 \Rightarrow \vec{a}\cdot\vec{b} = 0$ ...(ii)

Now, taking dot product with $\vec{a}$ in (i), we get

$$\vec{a}\cdot\vec{a} = (2\cos\theta)\,\vec{b}\cdot\vec{a} - \vec{c}\cdot\vec{a}$$

$\Rightarrow\ (\sqrt{2})^2 = 2\cos\theta \times 0 - \vec{a}\cdot\vec{c}$ [Using (ii)]

$\Rightarrow\ \vec{a}\cdot\vec{c} = -2$ ...(iii)

Again, taking dot product with $\vec{c}$ in (i), we get

$$\vec{a}\cdot\vec{c} = (2\cos\theta)\,(\vec{b}\cdot\vec{c}) - \vec{c}\cdot\vec{c}$$

$\Rightarrow\ -2 = 2\cos\theta\cdot2\cos\theta - 4$

$\Rightarrow\ 4\cos^2\theta = 2 \Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$ ($\because\ 0 < \theta < \pi/2$)

$\Rightarrow\ \theta = 45°$

$\therefore\ 1 + \tan 45° = 2$

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beat the
TIME TRAP

Duration : 30 minutes

SECTION-I

Single Option Correct Type

1. A point P moves so that the points A, B, C on x, y, z axes are such that PA, PB, PC are mutually perpendicular and volume of the tetrahedron $OABC$ is k. The locus of P is $(x^2 + y^2 + z^2)^3 =$

(a) $12\,kxyz$ (b) $24\,kxyz$
(c) $48\,kxyz$ (d) $6\,kxyz$

2. The distance of the origin from the normal at y - axis to the curve $y = (1 + x)^y + \sin^{-1}(\sin^2 x)$ is

(a) 1 (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

3. If p, q are chosen randomly with replacement from the set $\{1, 2, 3, \ldots, 10\}$, the probability, that the roots of the equation $x^2 + px + q = 0$ are real, is

(a) $\frac{3}{5}$ (b) $\frac{31}{50}$ (c) $\frac{61}{100}$ (d) $\frac{29}{50}$

4. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$, where $pq \neq 0$, are bisected by the x-axis, then

(a) $p^2 = q^2$ (b) $p^2 = 8q^2$
(c) $p^2 < 8q^2$ (d) $p^2 > 8q^2$

5. $\lim\limits_{n \to \infty} \prod\limits_{r=2}^{n} \left(\frac{r^3+1}{r^3-1}\right) =$

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 2

6. Let $x, y \in R$. If $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$, then $x + y =$

(a) 1 (b) 2 (c) -1 (d) -3

7. If $f(x) = \cos(\ln x)$, then $f(x)f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right] =$

(a) -1 (b) $\frac{1}{2}$ (c) -2 (d) 0

8. $\int_0^1 \tan^{-1}(1 - x + x^2)\,dx =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\ln 2$ (d) $\frac{\pi}{2} - \ln 2$

9. The curves $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively, the areas of these parts numbered from top to bottom, then $S_1 : S_2 : S_3 =$

(a) $1 : 2 : 3$ (b) $1 : 2 : 1$
(c) $1 : 1 : 1$ (d) $2 : 1 : 2$

10. A unit vector $\vec{d}$ is equally inclined at an angle α with the vectors $\vec{a} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$, $\vec{b} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$, $\vec{c} = \hat{k}$. Then $\alpha =$

(a) $\cos^{-1}\frac{1}{\sqrt{3}}$ (b) $\cos^{-1}\frac{1}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

SECTION-II

Numerical Answer Type

11. If $\int \sin 4x\, e^{\tan^2 x}\,dx = a\cos^b x\, e^{\tan^2 x} + K$, then the value of $a + b$ is

12. If the pair of lines $x^2(\cos^2\theta + \tan^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ make angles α and β with the x-axis, then the difference of their tangents is ____.

13. If the integer k is added to each of the numbers 36, 300, 596, one obtains the squares of three consecutive terms of an A.P., then the last digit of k is

14. If $2^{4n} - 15n - 1, n \in N$, is divisible by 15^k, the largest value of k is

15. The number of roots of $\cos^{-1}x + \cos^{-1}2x = \frac{\pi}{2}$ is

SOLUTIONS

1. (c) : $A(\alpha, 0, 0), B(0, \beta, 0), C(0, 0, \gamma)$ and $P(x, y, z)$.

Volume of tetrahedron $OABC = k$

$$\therefore \quad \frac{1}{6}\alpha\beta\gamma = k \Rightarrow \alpha\beta\gamma = 6k \qquad \text{...(A)}$$

$PA \perp PB \Rightarrow x(x - \alpha) + y(y - \beta) + z^2 = 0$

or $\quad x^2 + y^2 + z^2 = \alpha x + \beta y \qquad$ ...(i)

Likewise, $x^2 + y^2 + z^2 = \beta y + \gamma z \qquad$ ...(ii)

$x^2 + y^2 + z^2 = \gamma z + \alpha x \qquad$ ...(iii)

Adding, $\alpha x + \beta y + \gamma z = (x^2 + y^2 + z^2)\frac{3}{2} \qquad$ ...(iv)

$$\text{(i) – (iv)} \Rightarrow \alpha x = \beta y = \gamma z = \frac{x^2 + y^2 + z^2}{2} \qquad \text{...(B)}$$

From, (A) and (B), we have $\frac{(x^2 + y^2 + z^2)^3}{8xyz} = 6k$

or $\quad (x^2 + y^2 + z^2)^3 = 48xyzk$.

2. (d) : $x = 0 \Rightarrow y = 1$. Differentiating the given relation, we get

$$y' = (1 + x)^y\left(\frac{y}{1+x} + y'\ln(1+x)\right) + \frac{2\sin x\cos x}{\sqrt{1-\sin^4 x}}, \ y'(0) = 1$$

The equation of the normal is $(y - 1) + (x - 0) = 0$

$\Rightarrow \quad x + y - 1 = 0$

$\therefore$ The distance of the normal from origin is $\frac{1}{\sqrt{2}}$.

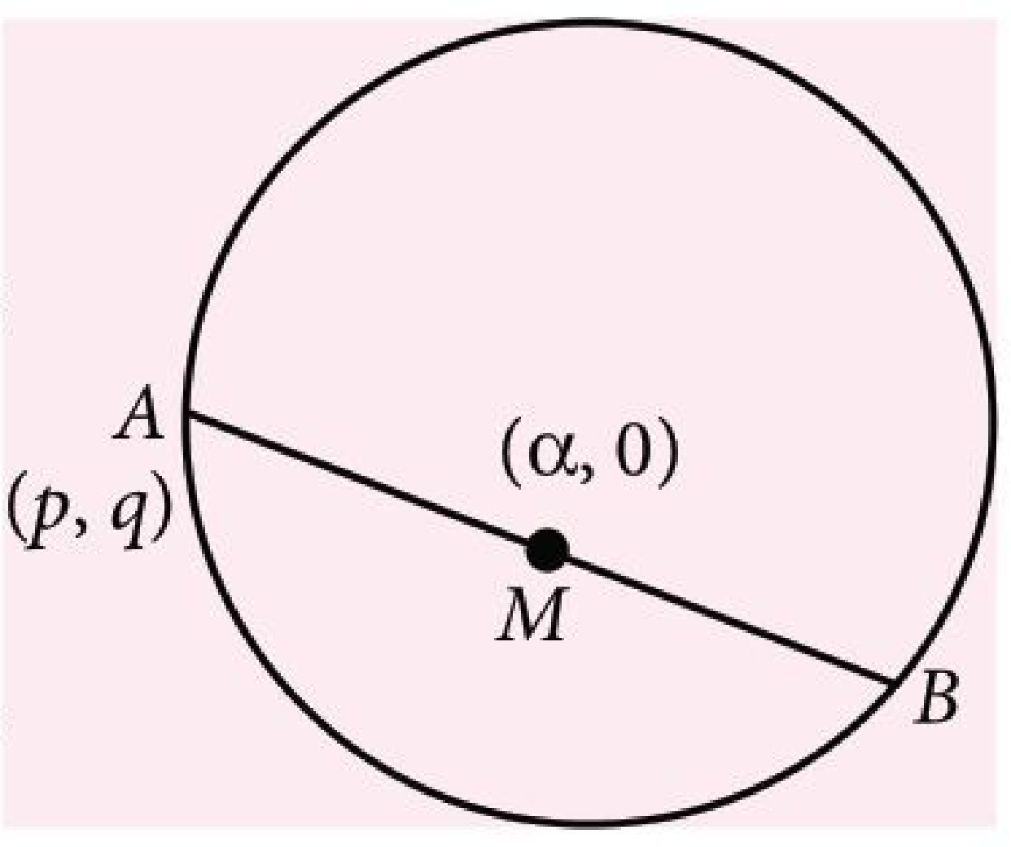
3. (b) : $p^2 \geq 4q$

p	2	3	4	5	6	7, 8, 9, 10
q	1	1, 2	1 to 4	1 to 6	1 to 9	4 times (1 to 10)

$\Rightarrow$ The total number of pairs (p, q) is

$1 + 2 + 4 + 6 + 9 + 40 = 62$

$$\therefore \quad \text{Required probability} = \frac{62}{10 \cdot 10} = \frac{31}{50}$$



4. (d) : $M(\alpha, 0)$ is on the x-axis.

M is the midpoint of AB.

$\therefore \quad B \equiv (2\alpha - p, -q)$ which lies on the circle

$x^2 + y^2 = px + qy$

$\therefore \quad (2\alpha - p)^2 + q^2 = p(2\alpha - p) - q^2$,

or $2\alpha^2 - 3p\alpha + p^2 + q^2 = 0$

As these are two chords, there are two values of α.

$\therefore \quad$ Discriminant > 0.

$\Rightarrow \quad 9p^2 - 8(p^2 + q^2) > 0$ or $p^2 > 8q^2$.

5. (c) : $\lim\limits_{n\to\infty}\prod\limits_{r=2}^{n}\frac{(r+1)(r^2-r+1)}{(r-1)(r^2+r+1)}$

$$= \lim_{n\to\infty}\prod_{r=2}^{n}\frac{r+1}{r-1}\cdot\prod_{r=2}^{n}\frac{r^2-r+1}{r^2+r+1}$$

$$= \lim_{n\to\infty}\frac{3}{1}\cdot\frac{4}{2}\cdot\frac{5}{3}\cdots\frac{n+1}{n-1}\cdot\frac{3}{7}\cdot\frac{7}{13}\cdots\frac{n^2-n+1}{n^2+n+1}$$

$$= \lim_{n\to\infty}\frac{1}{1\cdot 2}\cdot n(n+1)\cdot\frac{3}{n^2+n+1} = \lim_{n\to\infty}\frac{3}{2}\cdot\frac{1+\frac{1}{n}}{1+\frac{1}{n}+\frac{1}{n^2}} = \frac{3}{2}$$

6. (b) : $\frac{x+(x-2)i}{3+i} + \frac{2y + i(1 - 3y)}{3 - i} = i$

Multiplying by $(3 + i)(3 - i)$, we get

$(3 - i)(x + (x - 2)i) + (3 + i)(2y + i(1 - 3y)) = 10i$

Equating the real parts on both sides

$3x + x - 2 + 6y - (1 - 3y) = 0$

$\Rightarrow \quad 4x + 9y = 3 \qquad$ ...(i)

Equating the imaginary parts, we get

$-x + 3(x - 2) + 2y + 3(1 - 3y) = 10$

$\Rightarrow \quad 2x - 7y = 13 \qquad$ ...(ii)

Solving (i) and (ii), we get

$x = 3, y = -1, x + y = 2$.

7. (d) : $f(xy) = \cos\ln(xy) = \cos(\ln x + \ln y)$

Also, $f\left(\frac{x}{y}\right) = \cos\ln\left(\frac{x}{y}\right) = \cos(\ln x - \ln y)$

Now, $f(xy) + f\left(\frac{x}{y}\right) = \cos(\ln x + \ln y) + \cos(\ln x - \ln y)$

$= 2\cos\ln x \cdot \cos\ln y = 2f(x)f(y)$

$$\therefore \quad f(x)f(y) - \frac{1}{2}\left[f(xy) + f\left(\frac{x}{y}\right)\right] = 0$$

8. (c) : $\int\limits_0^1 \tan^{-1}(1-x+x^2)dx = \int\limits_0^1 \frac{\pi}{2} - \cot^{-1}(1-x+x^2)dx$

$$= \frac{\pi}{2} - \int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx = \frac{\pi}{2} - \int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right)dx$$

$$= \frac{\pi}{2} - \int_0^1 (\tan^{-1}x + \tan^{-1}(1-x))\,dx$$

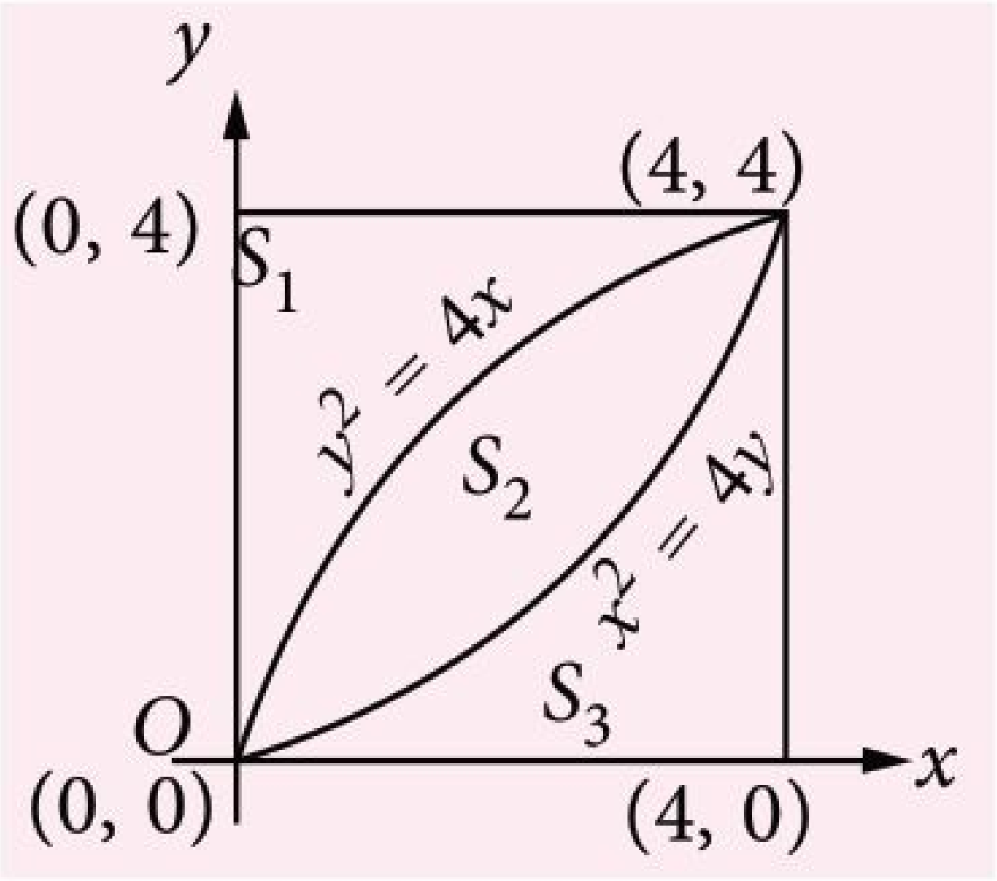
$$= \frac{\pi}{2} - \int_0^1 \tan^{-1}x\,dx - \int_0^1 \tan^{-1}(1-x)dx, \ x \to 1 - x$$

$$= \frac{\pi}{2} - 2\int_0^1 \tan^{-1} x\, dx$$

$$= \frac{\pi}{2} - 2\left[x\tan^{-1}x\Big|_0^1 - \int_0^1 \frac{x}{1+x^2}dx\right] = \int_0^1 \frac{2x}{1+x^2}dx$$

$$= \ln\left(1+x^2\right)\Big|_0^1 = \ln 2.$$

9. (c) : $S_2 = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right)dx$



$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$$

$$S_1 = S_3 = \frac{1}{2}\left(16 - \frac{16}{3}\right) = \frac{16}{3}$$

$\therefore\ S_1 : S_2 : S_3 = 1 : 1 : 1.$

10. (a) : $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\Rightarrow\ \vec{d}\cdot\vec{a} = \vec{d}\cdot\vec{b} = \vec{d}\cdot\vec{c} = \cos\alpha,\ \vec{c} = \hat{k}$

$\vec{d}\cdot(\vec{a} - \hat{k}) = 0,\ \vec{d}\cdot(\vec{b} - \hat{k}) = 0$

$\therefore$ $\vec{d}$ is along $(\vec{a} - \hat{k}) \times (\vec{b} - \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\theta & \sin\theta & -1 \\ -\sin\theta & \cos\theta & -1 \end{vmatrix}$$

$$= (\cos\theta - \sin\theta)\hat{i} + (\cos\theta + \sin\theta)\hat{j} + \hat{k}$$

$$\therefore\ \vec{d} = \frac{(\cos\theta - \sin\theta)\hat{i} + (\cos\theta + \sin\theta)\hat{j} + \hat{k}}{\sqrt{(\cos\theta - \sin\theta)^2 + (\cos\theta + \sin\theta)^2 + 1}}$$

$$= \frac{1}{\sqrt{3}}\left[(\cos\theta - \sin\theta)\hat{i} + (\cos\theta + \sin\theta)\hat{j} + \hat{k}\right]$$

$$\Rightarrow\ \cos\alpha = \vec{d}\cdot\hat{k} = \frac{1}{\sqrt{3}}.$$

11. (2) : Let $I = \int \sin 4x \cdot e^{\tan^2 x} dx$

$$I = 4\int \tan x \cdot \sec^2 x \cdot \cos^6 x\,(1 - \tan^2 x)\, e^{\tan^2 x} dx$$

Put $\tan^2 x = t \quad \Rightarrow \quad 2\tan x \sec^2 x\, dx = dt$

$$\therefore\ \ I = 2\int \frac{(1-t)\,e^t}{(1+t)^3}dt = -2\int \frac{(t+1-2)\,e^t}{(1+t)^3}dt$$

$$\Rightarrow I = \frac{-2e^t}{(1+t)^2} + K \Rightarrow I = -2\cos^4 x \cdot e^{\tan^2 x} + K$$

$\therefore\ \ a + b = 2$

12. (2) : If the slopes are m_1 and m_2 then

$$m_1 + m_2 = \frac{2\tan\theta}{\sin^2\theta},\ m_1 m_2 = \frac{\cos^2\theta + \tan^2\theta}{\sin^2\theta},$$

$$(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1 m_2$$

$$= \frac{4(\tan^2\theta - \sin^2\theta(\cos^2\theta + \tan^2\theta))}{\sin^4\theta},$$

$$= \frac{4(\sec^2\theta - \cos^2\theta - \tan^2\theta)}{\sin^2\theta} = 4.$$

13. (5) : Let the A.P. be $a,\ a + d,\ a + 2d$

$a^2 = 36 + k,\ (a + d)^2 = 300 + k,\ (a + 2d)^2 = 596 + k$

$\Rightarrow\ 2ad + d^2 = 300 - 36 = 264$ and

$2ad + 3d^2 = 596 - 300 = 296$

$d^2 = 16,\ d = 4,\ a = 31$

$\therefore\ \ k = a^2 - 36 = 31^2 - 36 = 925.$

14. (2) : $2^{4n} - 15n - 1 = 16^n - 15n - 1$

$= (1 + 15)^n - 15n - 1$

$$= \binom{n}{2}15^2 + \binom{n}{3}15^3 + \ldots.$$

$\therefore$ It is divisible by 15^2

15. (1) : $\cos^{-1} 2x = \frac{\pi}{2} - \cos^{-1} x = \sin^{-1} x$

$\therefore\ \ 2x = \cos(\sin^{-1} x) = \sqrt{1 - x^2}$

$$\Rightarrow\ 4x^2 = 1 - x^2 \Rightarrow x^2 = \frac{1}{5} \Rightarrow x = \pm\frac{1}{\sqrt{5}}$$

But $x = \frac{-1}{\sqrt{5}}$ makes each term of the left hand side greater than $\frac{\pi}{2}$.

$$\therefore\ \ x = \frac{1}{\sqrt{5}}.$$

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MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1. (c) **2.** (c) **3.** (c) **4.** (b) **5.** (c)
6. (b) **7.** (a,b,c) **8.** (b,c,d) **9.** (b,c)
10. (a,b,c,d) **11.** (a,c) **12.** (b,c) **13.** (a)
14. (a) **15.** (b) **16.** (a) **17.** (6) **18.** (2)
19. (5) **20.** (5)

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Let f be a twice differentiable function such that $f''(x) = -f(x), f'(x) = g(x)$ and $h(x) = [f(x)]^2 + [g(x)]^2$. Find $h(10)$, if $h(5) = 11$.

[Devika Aggarwal, Delhi]

Ans. Given, $h(x) = [f(x)]^2 + [g(x)]^2$

$$\Rightarrow \quad h'(x) = 2f(x)\cdot f'(x) + 2g(x)\cdot g'(x)$$

$$= 2[f(x)\cdot g(x) - g(x)\cdot f(x)] = 0$$

$$[\because f'(x) = g(x) \text{ and } g'(x) = f''(x) = -f(x)]$$

$$\therefore \quad h'(x) = 0 \ \forall\, x$$

$\therefore$ $h(x)$ is constant. $\Rightarrow$ $h(10) = h(5) = 11$.

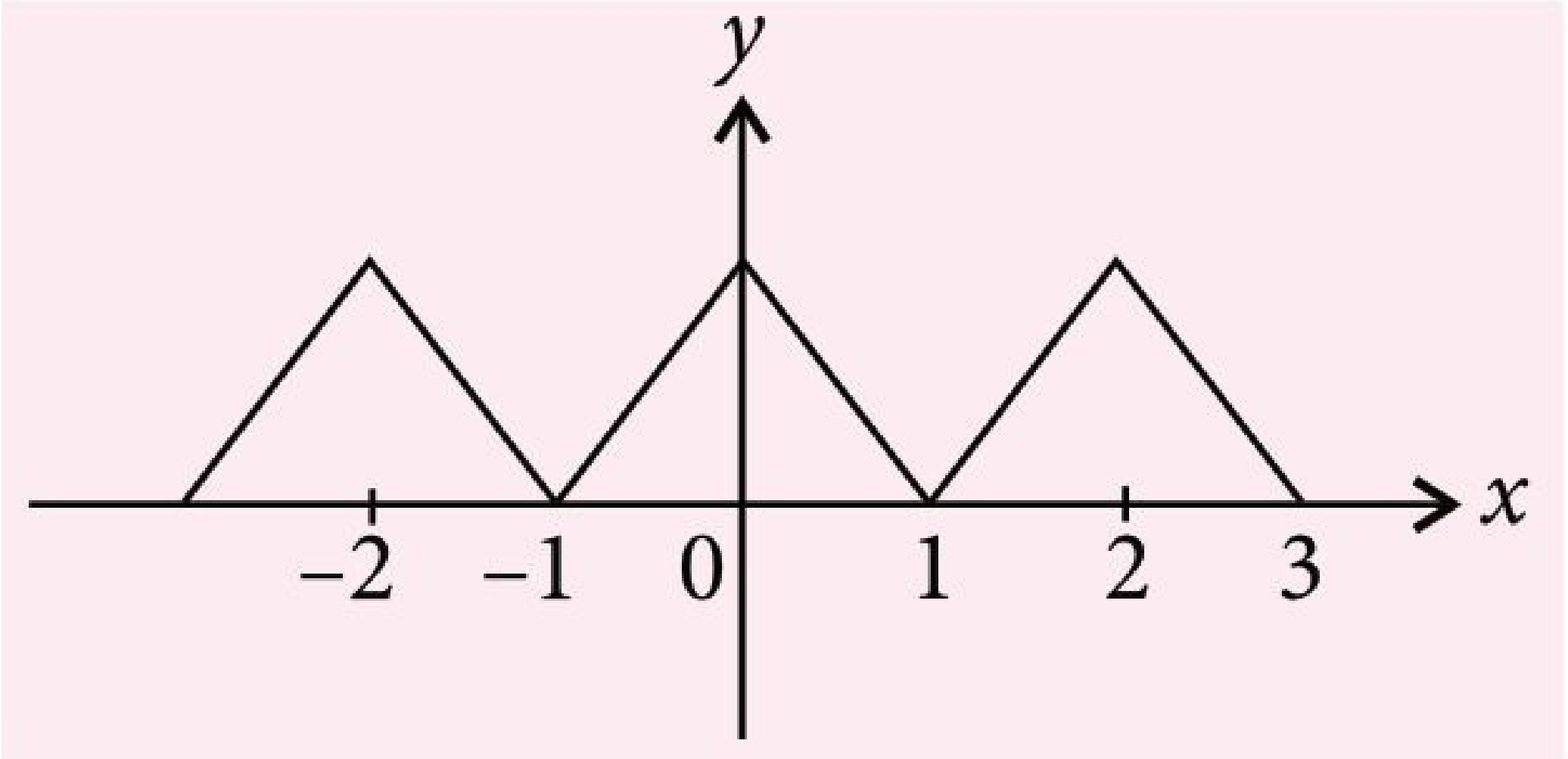
2. For any real number x, let $|x|$ denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval $[-10, 10]$ by

$$f(x) = \begin{cases} x - [x] & , \text{ if } [x] \text{ is odd} \\ 1 + [x] - x, & \text{if } [x] \text{ is even} \end{cases}$$

Then find the value of $\dfrac{\pi^2}{10}\displaystyle\int_{-10}^{10} f(x)\cos \pi x\, dx$.

[Ujjwal, Patna]

Ans. $f(x) = \begin{cases} \{x\}, & 2n-1 \le x < 2n \\ 1-\{x\}, & 2n \le x < 2n+1 \end{cases}$



We have $f(x)$ is periodic

$$I = \int_{-10}^{10} f(x)\cos \pi x\, dx = 2\int_0^{10} f(x)\cos \pi x\, dx$$

$$I = 2\times 5\int_0^2 f(x)\cos \pi x\, dx$$

$$I = 10\left[\int_0^1 (1-x)\cos \pi x\, dx + \int_1^2 (x-1)\cos \pi x\, dx\right]$$

$$= 10\,[(I_1) + (I_2)]$$

$$I_2 = \int_1^2 (x-1)\cos \pi x\, dx \ ; \ I_2 = -\int_0^1 t\cos \pi t\, dt \quad (x-1=t)$$

We have $I_1 = \displaystyle\int_0^1 (1-x)\cos \pi x\, dx = -\int_0^1 x\cos \pi x\, dx$

$$I = 10\left(-2\int_0^1 x\cos \pi x\, dx\right)$$

$$= -20\left[\frac{x\sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right]_0^1 = -20\left[-\frac{1}{\pi^2} - \frac{1}{\pi^2}\right] = \frac{40}{\pi^2}$$

Hence, $\dfrac{\pi^2}{10}\displaystyle\int_{-10}^{10} f(x)\cos \pi x\, dx = \frac{\pi^2}{10}\times\frac{40}{\pi^2} = 4$

3. Find the sum of the series, $\displaystyle\sum_{r=0}^{n} (-1)^r\, {}^nC_r \left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} + \ldots \text{upto } m \text{ terms}\right]$

[Cheenu, Bareilly]

Ans. $\displaystyle\sum_{r=0}^{n} (-1)^r\, {}^nC_r$

$$\left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \frac{15^r}{2^{4r}} + \ldots \text{ upto } m \text{ terms}\right]$$

$$= \sum_{r=0}^{n} (-1)^r\, {}^nC_r\left(\frac{1}{2}\right)^r + \sum_{r=0}^{n} (-1)^r\, {}^nC_r\left(\frac{3}{4}\right)^r + \sum_{r=0}^{n} (-1)^r\, {}^nC_r\left(\frac{7}{8}\right)^r + \ldots \text{ upto } m \text{ terms}$$

$$= \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \left(1-\frac{7}{8}\right)^n + \ldots \text{ upto } m \text{ terms}$$

$$\left\{\text{Using } \sum_{r=0}^{n} (-1)^r\, {}^nC_r x^r = (1-x)^n\right\}$$

$$= \left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n + \left(\frac{1}{8}\right)^n + \ldots \text{ upto } m \text{ terms}$$

$$= \frac{1}{2^n}\left(1 + \frac{1}{2^n} + \frac{1}{4^n} + \ldots \text{ upto } m \text{ terms}\right)$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{1-\left(\frac{1}{2^n}\right)^m}{1-\frac{1}{2^n}}\right] = \frac{2^{mn}-1}{2^{mn}(2^n-1)}.$$

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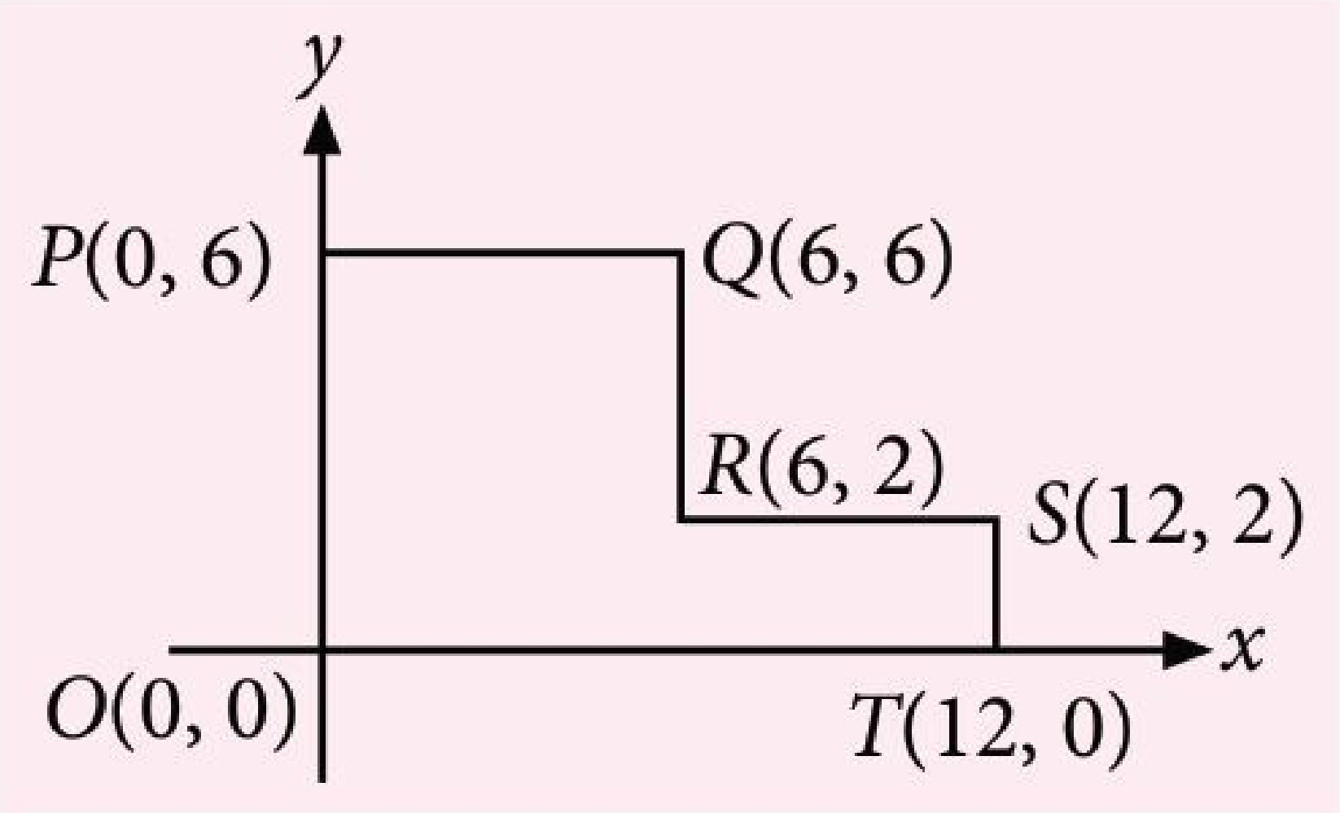
OLYMPIAD CORNER

1. In the diagram, two straight lines are to be drawn through $O(0, 0)$ so that the lines divide the figure $OPQRST$ into 3 pieces of equal area. Find the sum of the slopes of the lines.



2. Determine all linear functions $f(x) = ax + b$ such that if $g(x) = f^{-1}(x)$ for all values of x, then $f(x) - g(x) = 44$ for all values of x.
   (Note : f^{-1} is the inverse function of f)

3. (a) Determine all pairs (a, b) of positive integers for which $a^3 + 2ab = 2013$.
   (b) Determine all real values of x for which $\log_2(2^{x-1} + 3^{x+1}) = 2x - \log_2(3^x)$.

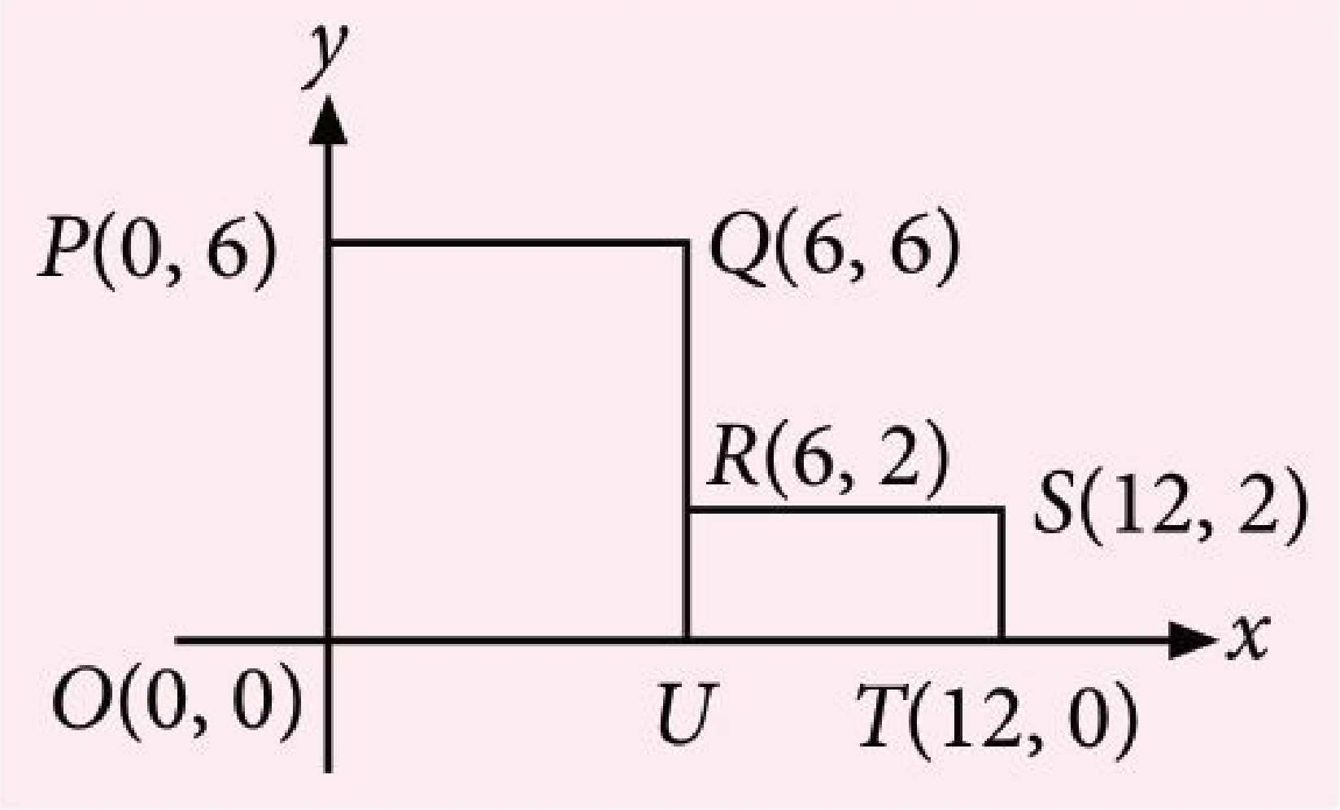
4. Integers can be written in bases other than the usual base 10. For example, the notation $(235)_7$ stands for the base 7 representation of the integer $2 \times 7^2 + 3 \times 7 + 5$ (which equals 124 in base 10).
   In general, if x, y and z are integers between 0 and $b - 1$, inclusive, then $(xyz)_b = xb^2 + yb + z$.
   Find all triples (x, y, z) for which $(xyz)_{10} = 2(xyz)_7$, where each of x, y and z comes from the list 1, 2, 3, 4, 5, 6.

5. Vernon starts with a first number n with $0 < n < 1$. Vernon enters the first number into a machine to produce a second number. He then enters the second number back into the machine to produce a third number. Vernon continually enters the result back into the machine giving a chain of numbers. When Vernon enters a number x into the machine,
   - if $x \le \frac{1}{2}$, the machine outputs $2x$, and
   - if $x > \frac{1}{2}$, the machine outputs $2(1 - x)$.

   If the machine ever produces the number 1, Vernon stops the process.
   (a) A chain starts with $\frac{3}{11}$. This gives
   $$\frac{3}{11} \to \frac{6}{11} \to \frac{10}{11} \to \frac{2}{11} \to \dots .$$
   What are the next four numbers in this chain?
   (b) Vernon enters a number x with $0 < x < 1$ into the machine and the machine produces the number x. Determine the value of x.
   (c) The fourth number in a chain is 1. Determine all of the possible values of the first number in this chain.

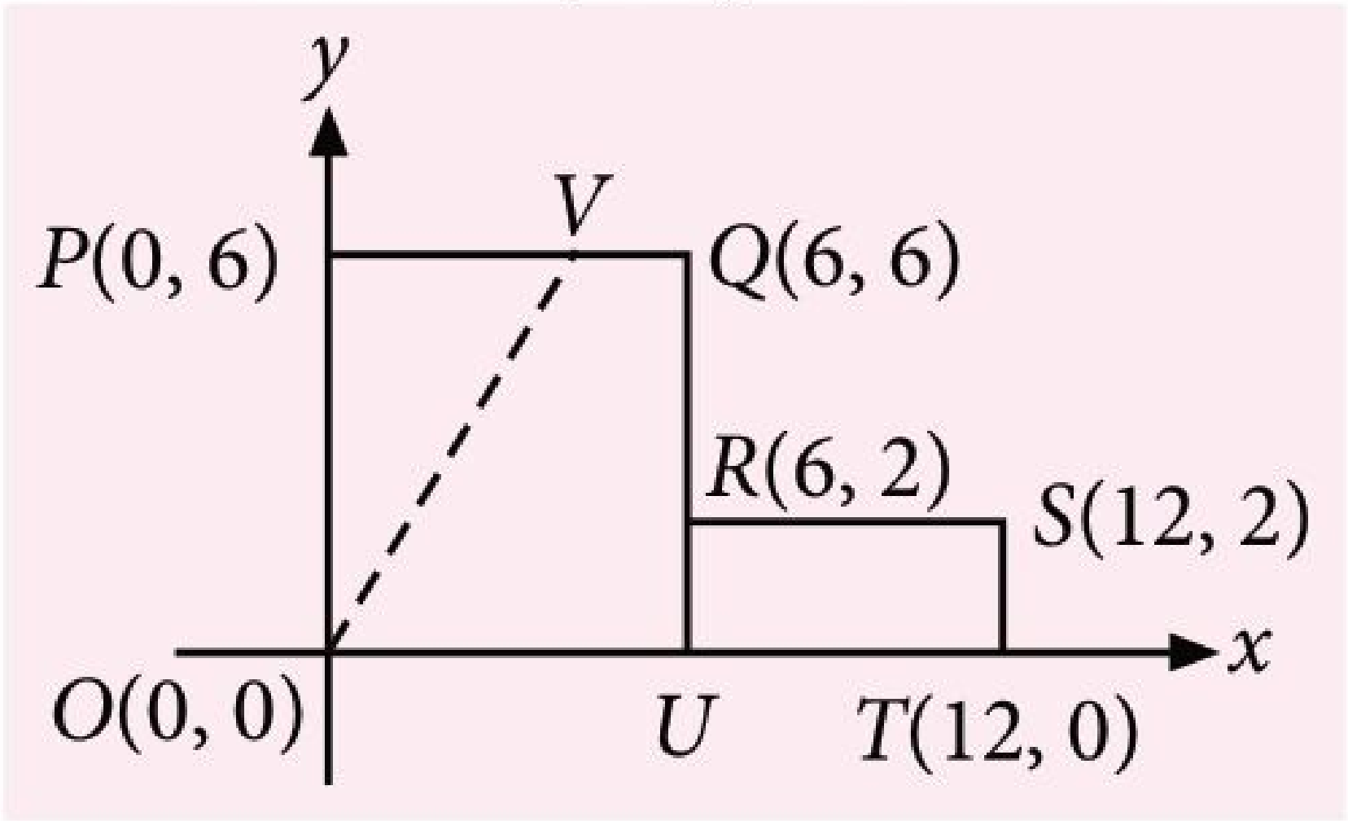
SOLUTIONS

1. Extend QR downwards to meet the x-axis at $U(6, 0)$.



The area of figure $OPQRST$ equals the sum of the areas of square $OPQU$ (which has side length 6, so area 36) and rectangle $RSTU$ (which has height 2 and width 6, so area 12).
Thus, the area of figure $OPQRST$ is 48. If we are to divide the figure into three pieces of equal area, then each piece has area 16.
Let V be the first point on the perimeter (measured clockwise from P) so that the line through O and V cuts off an area of 16.
Note that the area of ΔOPQ is half of the area of square $OPQU$, or 18, so V is to the left of Q on PQ.

Thus, V has coordinates $(v, 6)$ for some number v.



Consider ΔOPV having base OP of length 6 and height PV of length v.

Since, the area of ΔOPV is 16, then

$\frac{1}{2}(6)(v) = 16$ or $3v = 16$ or $v = \frac{16}{3}$.

Therefore, the slope of OV is $\frac{6}{\frac{16}{3}} = \frac{9}{8}$.

Let W be the second desired point.

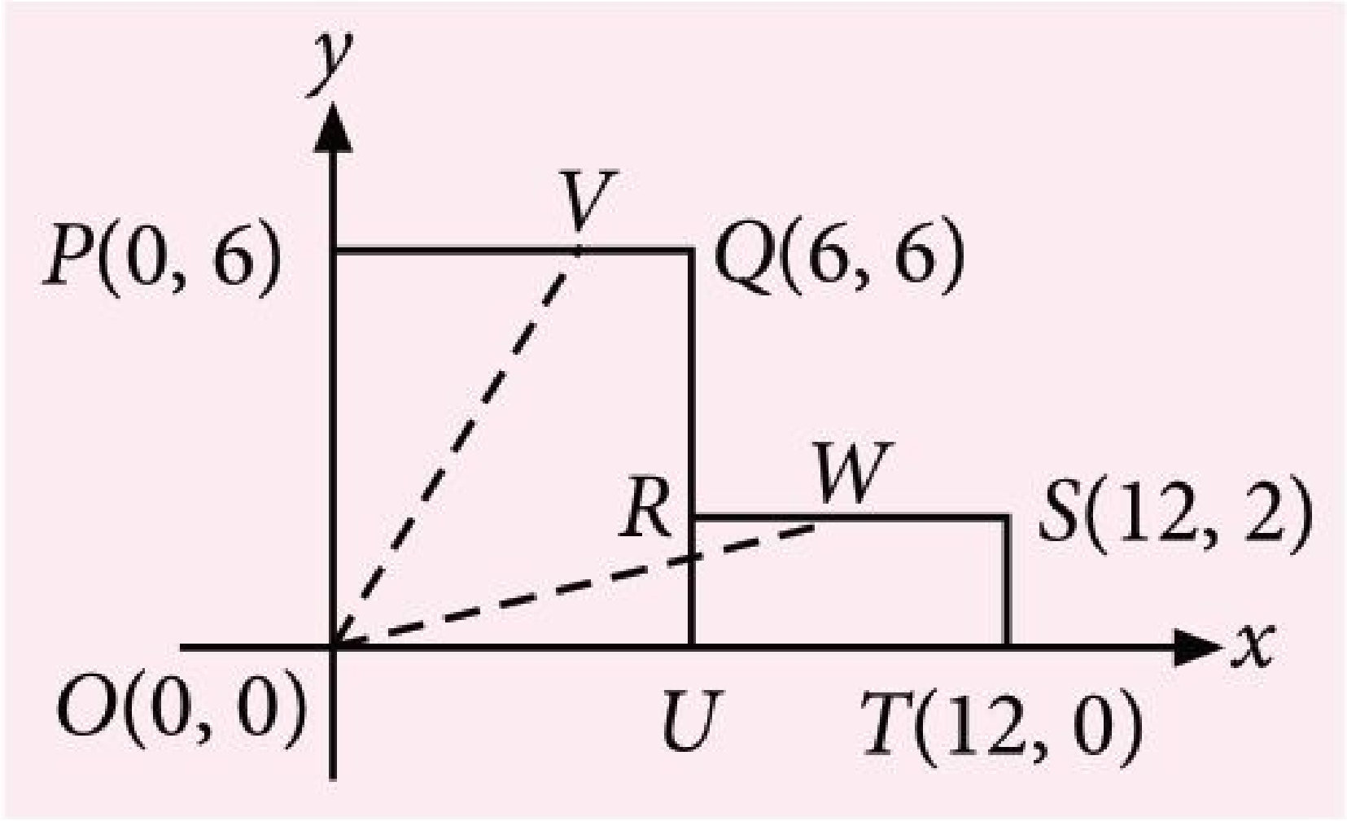
Since the area of $\Delta OTS = \frac{1}{2}(OT)(TS)$

$= \frac{1}{2}(12)(2) = 12$ (less than $\frac{1}{3}$ of the total area)

and the area of trapezoid $ORST$ is $\frac{1}{2}(RS + OT)(ST)$

$= \frac{1}{2}(6 + 12)(2) = 18$

(more than $\frac{1}{3}$ of the total area), then W lies on RS.



Suppose that W has coordinates $(w, 2)$ for some number w.

We want the area of trapezoid $WSTO$ to be 16.

Therefore, $\frac{1}{2}(WS + OT)(ST) = 16$

$\Rightarrow \frac{1}{2}(12 - w + 12)(2) = 16$

$\Rightarrow 24 - w = 16 \Rightarrow w = 8$

Thus, the coordinates of W are $(8, 2)$, and so the slope of OW is $\frac{2}{8} = \frac{1}{4}$.

Thus, the sum of the two required slopes is $\frac{9}{8} + \frac{1}{4} = \frac{11}{8}$.

2. Since $f(x) = ax + b$, we can determine an expression for $g(x) = f^{-1}(x)$ by letting $y = f(x)$ and to obtain $y = ax + b$. We then interchange x and y to obtain $x = ay + b$ which we solve for y to obtain $ay = x - b$ or $y = \frac{x}{a} - \frac{b}{a}$.

Therefore, $f^{-1}(x) = \frac{x}{a} - \frac{b}{a}$.

Note that $a \neq 0$. (This makes sense since the function $f(x) = b$ has a graph which is a horizontal line, and so cannot be invertible.)

Therefore, the equation $f(x) - g(x) = 44$ becomes

$(ax + b) - \left(\frac{x}{a} - \frac{b}{a}\right) = 44$ or $\left(a - \frac{1}{a}\right)x + \left(b + \frac{b}{a}\right) = 44$

$= 0x + 44$, and this equation is true for all x.

Since the equation $\left(a - \frac{1}{a}\right)x + \left(b + \frac{b}{a}\right) = 0x + 44$ is true for all x, then the coefficients of the linear expression on the left side must match the coefficients of the linear expression on the right side.

Therefore, $a - \frac{1}{a} = 0$ and $b + \frac{b}{a} = 44$.

From the first of these equations, we obtain $a = \frac{1}{a}$ or $a^2 = 1$, which gives $a = 1$ or $a = -1$.

If $a = 1$, the equation $b + \frac{b}{a} = 44$ becomes $b + b = 44$, which gives $b = 22$.

If $a = -1$, the equation $b + \frac{b}{a} = 44$ becomes $b - b = 44$, which is not possible.

Therefore, we must have $a = 1$ and $b = 22$, and so $f(x) = x + 22$.

3. (a) First, we factor the left side of the given equation to obtain $a(a^2 + 2b) = 2013$.

Next, we factor the integer 2013 as $2013 = 3 \times 671 = 3 \times 11 \times 61$.

Note that each of 3, 11 and 61 is prime, so we can factor 2013 no further (We can find the factors of 3 and 11 using tests for divisibility by 3 and 11, or by systematic trial and error.)

Since $2013 = 3 \times 11 \times 61$, then the positive divisors of 2013 are 1, 3, 11, 33, 61, 183, 671, 2013

Since, a and b are positive integers, then a and $a^2 + 2b$ are both positive integers.

Since a and b are positive integers, then $a^2 \geq a$ and $2b > 0$, so $a^2 + 2b > a$. Since $a(a^2 + 2b) = 2013$, then a and $a^2 + 2b$ must be a divisor pair of 2013 (that is, a pair of positive integers whose product is 2013) with $a < a^2 + 2b$.

We make a table of the possibilities:

a	$a^2 + 2b$	$2b$	b
1	2013	2012	1006
3	671	662	331
11	183	62	31
33	61	–1028	N/A

Note that the last case is not possible, since b must be positive.

Therefore, the three pairs of positive integers that satisfy the equation are (1, 1006), (3, 331), (11, 31). (We can verify by substitution that each is a solution of the original equation.)

(b) We successively manipulate the given equation to produce equivalent equations :

$\log_2(2^{x-1} + 3^{x+1}) = 2x - \log_2(3^x)$

$\Rightarrow \log_2(2^{x-1} + 3^{x+1}) + \log_2(3^x) = 2x$

$\Rightarrow \log_2((2^{x-1} + 3^{x+1})3^x) = 2x$

(Using $\log_2 A + \log_2 B = \log_2 AB$)

$\Rightarrow (2^{x-1} + 3^{x+1})3^x = 2^{2x}$ (Exponentiating both sides)

$\Rightarrow 2^{-1}2^x3^x + 3^13^x3^x = 2^{2x}$

$\Rightarrow \frac{1}{2}\cdot 2^x3^x + 3\cdot 3^{2x} = 2^{2x}$

$\Rightarrow 2^x3^x + 6\cdot 3^{2x} = 2\cdot 2^{2x}$ (multiplying by 2)

$\Rightarrow 2^x3^x2^{-2x} + 6\cdot 3^{2x}2^{-2x} = 2$

(dividing both sides by $2^{2x} \neq 0$)

$\Rightarrow 2^{-x}3^x + 6\cdot 3^{2x}\cdot 2^{-2x} = 2$

$\left(\frac{3}{2}\right)^x + 6\left(\frac{3}{2}\right)^{2x} = 2$

Next, we make the substitution $t = \left(\frac{3}{2}\right)^x$, noting that $\left(\frac{3}{2}\right)^{2x} = \left(\left(\frac{3}{2}\right)^x\right)^2 = t^2$.

Thus, we obtain the equivalent equations

$t + 6t^2 = 2$

$\Rightarrow 6t^2 + t - 2 = 0 \Rightarrow (3t + 2)(2t - 1) = 0$

Therefore, $t = -\frac{2}{3}$ or $t = \frac{1}{2}$.

Since $t = \left(\frac{3}{2}\right)^x > 0$, then we must have $t = \left(\frac{3}{2}\right)^x = \frac{1}{2}$.

Thus, $x = \log_{3/2}(1/2) = \frac{\log(1/2)}{\log(3/2)} = \frac{\log 1 - \log 2}{\log 3 - \log 2}$

$= \frac{-\log 2}{\log 3 - \log 2} = \frac{\log 2}{\log 2 - \log 3}$

4. Since, we are told that $(xyz)_b = xb^2 + yb + z$, then $(xyz)_{10} = 10^2x + 10y + z = 100x + 10y + z$ and $(xyz)_7 = 7^2x + 7y + z = 49x + 7y + z$.

From the given information

$$\begin{aligned} (xyz)_{10} &= 2(xyz)_7 \\ 100x + 10y + z &= 2(49x + 7y + z) \\ 100x + 10y + z &= 98x + 14y + 2z \\ 2x &= 4y + z \end{aligned}$$

Since the left side of this equation is an even integer ($2x$), then the right side must also be an even integer. Since $4y$ is an even integer, then for $4y + z$ to be an even integer, it must be the case that z is an even integer.

This gives us three possibilities: $z = 2$, $z = 4$ and $z = 6$.

Case I : $z = 2$

Here, $2x = 4y + 2$ or $x = 2y + 1$.

We try the possible values for y :

- $y = 1$ gives $x = 2(1) + 1 = 3$; this gives the triple $(x, y, z) = (3, 1, 2)$
- $y = 2$ gives $x = 2(2) + 1 = 5$; this gives the triple $(x, y, z) = (5, 2, 2)$
- If y is at least 3, then $x = 2y + 1$ is at least 7, which is impossible.

Therefore, there are two triples that work when $z = 2$.

Case II : $z = 4$

Here, $2x = 4y + 4$ or $x = 2y + 2$.

We try the possible values for y :

- $y = 1$ gives $x = 2(1) + 2 = 4$; this gives the triple $(x, y, z) = (4, 1, 4)$
- $y = 2$ gives $x = 2(2) + 2 = 6$; this gives the triple $(x, y, z) = (6, 2, 4)$
- If y is at least 3, then $x = 2y + 2$ is at least 8, which is impossible.

Therefore, there are two triples that work when $z = 4$.

Case III : $z = 6$

Here, $2x = 4y + 6$ or $x = 2y + 3$.

We try the possible values for y :

- $y = 1$ gives $x = 2(1) + 3 = 5$; this gives the triple $(x, y, z) = (5, 1, 6)$
- If y is at least 2, then $x = 2y + 3$ is at least 7, which is impossible.

Therefore, there is one triple that works when $z = 6$.

Finally, the triples that satisfy the equation are $(x, y, z) = (3, 1, 2), (5, 2, 2), (4, 1, 4), (6, 2, 4), (5, 1, 6)$.

5. **(a)** Since the fourth number in the chain is 2/11 and 2/11 is less than 1/2, then the fifth number is $2\left(\frac{2}{11}\right)=\frac{4}{11}$.

Since the fifth number in the chain is $\frac{4}{11}$ and $\frac{4}{11}$ is less than $\frac{1}{2}$, then the sixth number is $2\left(\frac{4}{11}\right)=\frac{8}{11}$.

Since the sixth number in the chain is $\frac{8}{11}$ and $\frac{8}{11}$ is larger than $\frac{1}{2}$, then the seventh number is $2\left(1-\frac{8}{11}\right)=2\left(\frac{3}{11}\right)=\frac{6}{11}$.

Since the seventh number in the chain is $\frac{6}{11}$ and $\frac{6}{11}$ is larger than $\frac{1}{2}$, then the eighth number is $2\left(1-\frac{6}{11}\right)=2\left(\frac{5}{11}\right)=\frac{10}{11}$.

Therefore, the next four numbers in the chain are $\frac{4}{11}, \frac{8}{11}, \frac{6}{11}, \frac{10}{11}$.

(b) Therefore are two possibilities :

$x \le \frac{1}{2}$ and $x > \frac{1}{2}$.

If $x \le \frac{1}{2}$ and x is entered into the machine, then the machine outputs $2x$.

Since the input is to be the same as the output, then $x = 2x$ or $x = 0$.

This is impossible since $x > 0$.

If $x > \frac{1}{2}$ and x is entered into the machine, then the machine outputs $2(1 - x)$.

Since the input is to be the same as the output, then $x = 2(1 - x)$ or $x = 2 - 2x$.

Thus, $3x = 2$ and so $x = \frac{2}{3}$, which satisfies the restrictions.

Therefore, if x is entered into the machine and x is produced, then $x = \frac{2}{3}$.

(c) Suppose that the chain is $a, b, c, 1$.

In this part, we have to produce the chain backwards (that is, we have to determine c, b and a).

We make the following general observation:

Suppose that x is entered into the machine and y is produced.

If $x \le \frac{1}{2}$, then $y = 2x$.

Solving for x in terms of y, we obtain $x=\frac{1}{2}y$.

(This gives us the input in terms of the output)

If $x > \frac{1}{2}$, then $y = 2(1 - x)$.

Solving for x in terms of y, we obtain $y = 2 - 2x$ or $2x = 2 - y$ or $x=\frac{1}{2}(2-y)$.

Since the third number in the chain is c and the fourth is 1, then from above, $c=\frac{1}{2}(1)=\frac{1}{2}$ or $c=\frac{1}{2}(2-1)=\frac{1}{2}$.

In either case, the chain is $a, b, \frac{1}{2}, 1$.

Since the second number in the chain is b and the third is $\frac{1}{2}$, then from above,

$$b=\frac{1}{2}\left(\frac{1}{2}\right)=\frac{1}{4} \text{ or } b=\frac{1}{2}\left(2-\frac{1}{2}\right)=\frac{1}{2}\left(\frac{3}{2}\right)=\frac{3}{4}.$$

Therefore, the chain is either $a, \frac{1}{4}, \frac{1}{2}, 1$ or $a, \frac{3}{4}, \frac{1}{2}, 1$. If the chain is $a, \frac{1}{4}, \frac{1}{2}, 1$, then the first number is a and the second number is $\frac{1}{4}$.

Thus, either $a=\frac{1}{2}\left(\frac{1}{4}\right)=\frac{1}{8}$ or

$$a=\frac{1}{2}\left(2-\frac{1}{4}\right)=\frac{1}{2}\left(\frac{7}{4}\right)=\frac{7}{8}.$$

If the chain is $a, \frac{3}{4}, \frac{1}{2}, 1$, then the first number is a and the second number is $\frac{3}{4}$.

Thus, either $a=\frac{1}{2}\left(\frac{3}{4}\right)=\frac{3}{8}$

or $a=\frac{1}{2}\left(2-\frac{3}{4}\right)=\frac{1}{2}\left(\frac{5}{4}\right)=\frac{5}{8}$.

Therefore, the possible first numbers in the chain are $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}$.

We can double check that each of these gives a fourth number of 1:

$$\frac{1}{8}\to\frac{1}{4}\to\frac{1}{2}\to 1 \qquad \frac{3}{8}\to\frac{3}{4}\to\frac{1}{2}\to 1$$

$$\frac{5}{8}\to\frac{3}{4}\to\frac{1}{2}\to 1 \qquad \frac{7}{8}\to\frac{1}{4}\to\frac{1}{2}\to 1$$

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CBSE

Term-II

warm-up!

CLASS-XI

Practice Paper 2022

Time allowed : 2 hours
Maximum marks : 40

General Instructions :

1. *This question paper contains three sections - A, B and C. Each part is compulsory.*
2. *Section - A has 6 short answer type (SA1) questions of 2 marks each.*
3. *Section - B has 4 short answer type (SA2) questions of 3 marks each.*
4. *Section - C has 4 long answer type questions (LA) of 4 marks each.*
5. *There is an internal choice in some of the questions.*
6. *Q14 is a case-based problem having 2 sub parts of 2 marks each.*

SECTION - A

1. The angles of a triangle are in the ratio $3:4:5$. Find the smallest angle in degree and the greatest angle in radians.

2. Find the equation of set of points P such that $PA^2 + PB^2 = 2k^2$, where A and B are the points $(3, 4, 5)$ and $(-1, 3, -7)$, respectively.

3. If $\frac{n!}{5!(n-1)!}$ and $\frac{n!}{7!(n-3)!}$ are in the ratio 21 : 1, find the value of n.

OR

A polygon has 44 diagonals. If n denotes the number of vertices of polygon. Find the value of n. Hence find the number of triangles that can be formed by joining these n points.

4. The probability of an event A occurring is 0.5 and B occurring is 0.3. If A and B are mutually exclusive, then find the probability that neither A occur nor B.

5. Show that : $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\dfrac{1+\tan x}{1-\tan x}\right)^2$

6. Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.

SECTION - B

7. Differentiate $\dfrac{2x^3 - \sin x}{\cot x}$ w.r.t. x.

8. Find the equation of the circle having centre $(1, -2)$ and passing through the point of intersection of the lines $3x + y = 14$ and $2x + 5y = 18$.

OR

If a focal chord of the parabola $y^2 = ax$ is $2x - y - 8 = 0$, then find the equation of the directrix.

9. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of 8% solution, how many litres of 2% solution will have to be added?

OR

Solve the following system of inequalities graphically : $5x + 4y \le 40$, $x \ge 2$, $y \ge 3$.

10. Find the derivative of $\cos(5x + 2)$ w.r.t. x by first principle.

SECTION - C

11. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events :

A : the sum is greater than 8

B : 2 occurs on either die

C : the sum is at least 7 and a multiple of 3.

Which pairs of these events are mutually exclusive?

OR

(i) Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is neither divisible by 3 nor by 4?

(ii) What is the probability that the sum of the numbers on the two faces is divisible by 3 or 4?

12. Prove that :

$$\sin^3 x + \sin^3\left(\frac{2\pi}{3} + x\right) + \sin^3\left(\frac{4\pi}{3} + x\right) = -\frac{3}{4}\sin 3x$$

13. Find the equation of the ellipse whose foci are (2, 3), (–2, 3) and whose semi-minor axis is $\sqrt{5}$.

CASE-BASED/DATA-BASED

14. Sumit works at a book shop. While arranging some books on the book shelf, he observed that there are 5 History books, 3 Mathematics books and 4 Science books which are to be arranged on the shelf.

(i) If he selects 2 History books, 1 Maths book and 1 Science book to arrange them, then find the number of ways in which selection can be made.

(ii) Find the number of ways, if the books of same subject are put together.

SOLUTIONS

1. Let the three angles be $3x$, $4x$ and $5x$ degrees, then $3x + 4x + 5x = 180° \Rightarrow 12x = 180° \Rightarrow x = 15°$

$\therefore$ Smallest angle $3x = 45°$

and greatest angle $= 5x = 75°$

$$= 75 \times \frac{\pi}{180} \text{ radians} = \frac{5\pi}{12} \text{ radians}$$

2. Let the coordinates of point P be (x, y, z).

Here, $PA^2 = (x - 3)^2 + (y - 4)^2 + (z - 5)^2$

$PB^2 = (x + 1)^2 + (y - 3)^2 + (z + 7)^2$

By the given condition, $PA^2 + PB^2 = 2k^2$

$\Rightarrow (x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (z+7)^2 = 2k^2$

i.e., $2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z = 2k^2 - 109$.

3. Here, $\dfrac{n!}{5!(n-1)!} : \dfrac{n!}{7!(n-3)!} = 21:1$

$$\Rightarrow \frac{n!}{5!(n-1)!} \times \frac{7!(n-3)!}{n!} = \frac{21}{1}$$

$$\Rightarrow \frac{7!(n-3)!}{5!(n-1)!} = \frac{21}{1} \Rightarrow \frac{7 \times 6 \times 5!(n-3)!}{5!(n-1)(n-2)(n-3)!} = \frac{21}{1}$$

$$\Rightarrow \frac{42}{n^2 - 3n + 2} = \frac{21}{1}$$

$\Rightarrow n(n - 3) = 0 \Rightarrow n = 0$ or $n = 3$

But when $n = 0$, then $(n - 1)!$ and $(n - 3)!$ are not defined.

Thus, $n = 3$

OR

Here, number of vertices or sides $= n$

$\therefore$ Number of diagonals $= {}^nC_2 - n$

$$\Rightarrow 44 = {}^nC_2 - n \Rightarrow 44 = \frac{n(n-1)}{2} - n$$

$\Rightarrow n^2 - 3n - 88 = 0 \Rightarrow (n + 8)(n - 11) = 0$

$\Rightarrow n = 11$ or $n = -8$ (Rejected)

4. Here, $P(A) = 0.5$ and $P(B) = 0.3$

Now, $P(A \cup B) = P(A) + P(B)$ (Since, $A \cap B = \phi$)

$= 0.5 + 0.3 = 0.8$

$\therefore P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.8 = 0.2$

5. $$\text{L.H.S.} = \frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)}$$

$$= \left(\frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}\right) \div \left(\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}\right)$$

$$= \left(\frac{1 + \tan x}{1 - \tan x}\right) \div \left(\frac{1 - \tan x}{1 + \tan x}\right) = \left(\frac{1 + \tan x}{1 - \tan x}\right) \times \left(\frac{1 + \tan x}{1 - \tan x}\right)$$

$$= \left(\frac{1 + \tan x}{1 - \tan x}\right)^2 = \text{R.H.S.}$$

6. Let x be the smaller of the two consecutive odd natural numbers, so that the other one is $x + 2$. Then, we should have

$x > 10$ and $(x + 2) > 10$ ...(i)

and $x + (x + 2) < 40$

$\Rightarrow \quad 2x + 2 < 40 \Rightarrow 2x < 38$ *i.e.*, $x < 19$ ...(ii)

From (i) and (ii), we get $10 < x < 19$

Since x is an odd number, x can take the values 11, 13, 15, and 17. So, the required possible pairs will be (11, 13), (13, 15), (15, 17) and (17, 19).

7. Let $y = \dfrac{2x^3 - \sin x}{\cot x}$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left\{\frac{2x^3 - \sin x}{\cot x}\right\}$$

$$= \frac{\cot x \cdot \frac{d}{dx}(2x^3 - \sin x) - (2x^3 - \sin x)\cdot \frac{d}{dx}(\cot x)}{(\cot x)^2}$$

$$= \frac{\cot x \cdot (6x^2 - \cos x) - (2x^3 - \sin x)(-\text{cosec}^2 x)}{\cot^2 x}$$

$$= \frac{6x^2 \cot x - \cot x \cdot \cos x + 2x^3 \text{cosec}^2 x - \sin x \cdot \text{cosec}^2 x}{\cot^2 x}$$

$$= \frac{6x^2 \cdot \frac{\cos x}{\sin x} - \frac{\cos x}{\sin x} \cdot \cos x + 2x^3 \frac{1}{\sin^2 x} - \sin x \cdot \frac{1}{\sin^2 x}}{\frac{\cos^2 x}{\sin^2 x}}$$

$$= \frac{(6x^2 \cos x - \cos^2 x)\sin x + (2x^3 - \sin x)}{\cos^2 x}$$

8. The point of intersection of $3x + y - 14 = 0$ and $2x + 5y - 18 = 0$ is (4, 2).

Therefore, the radius $= \sqrt{9+16} = 5$ and hence the equation of the circle is given by

$(x - 1)^2 + (y + 2)^2 = 25$ or $x^2 + y^2 - 2x + 4y - 20 = 0$

OR

Equation of parabola is $y^2 = ax = 4 \cdot \dfrac{a}{4} x$ ...(i)

$\therefore$ Focus of parabola is $\left(\dfrac{a}{4}, 0\right)$.

Focal chord of the parabola is $2x - y - 8 = 0$ which passes through the focus.

$\therefore \; 2 \cdot \dfrac{a}{4} - 0 - 8 = 0 \Rightarrow a = 16$

Putting the value of a in (i), we get

$y^2 = 16x \Rightarrow y^2 = 4 \cdot 4x$

So, directrix is $x + 4 = 0$.

9. Let x litres of 2% solution be added.

$\therefore$ Total mixture $= (640 + x)$ litres

According to question, we get

4% of $(640 + x) <$ 2% of $x +$ 8% of $640 <$ 6% of $(640 + x)$

$\Rightarrow \quad 4(640 + x) < 2x + 8 \times 640 < 6\,(640 + x)$

$\Rightarrow \quad 2560 + 4x < 2x + 5120 < 3840 + 6x$

$\Rightarrow \quad 2560 + 4x < 2x + 5120$ and $2x + 5120 < 3840 + 6x$

$\Rightarrow \quad 2x < 2560$ and $4x > 1280$

$\Rightarrow \quad x < 1280$ and $x > 320$

$\therefore \quad 320 < x < 1280$

So, greater than 320 litres and less than 1280 litres of 2% solution be added.

OR

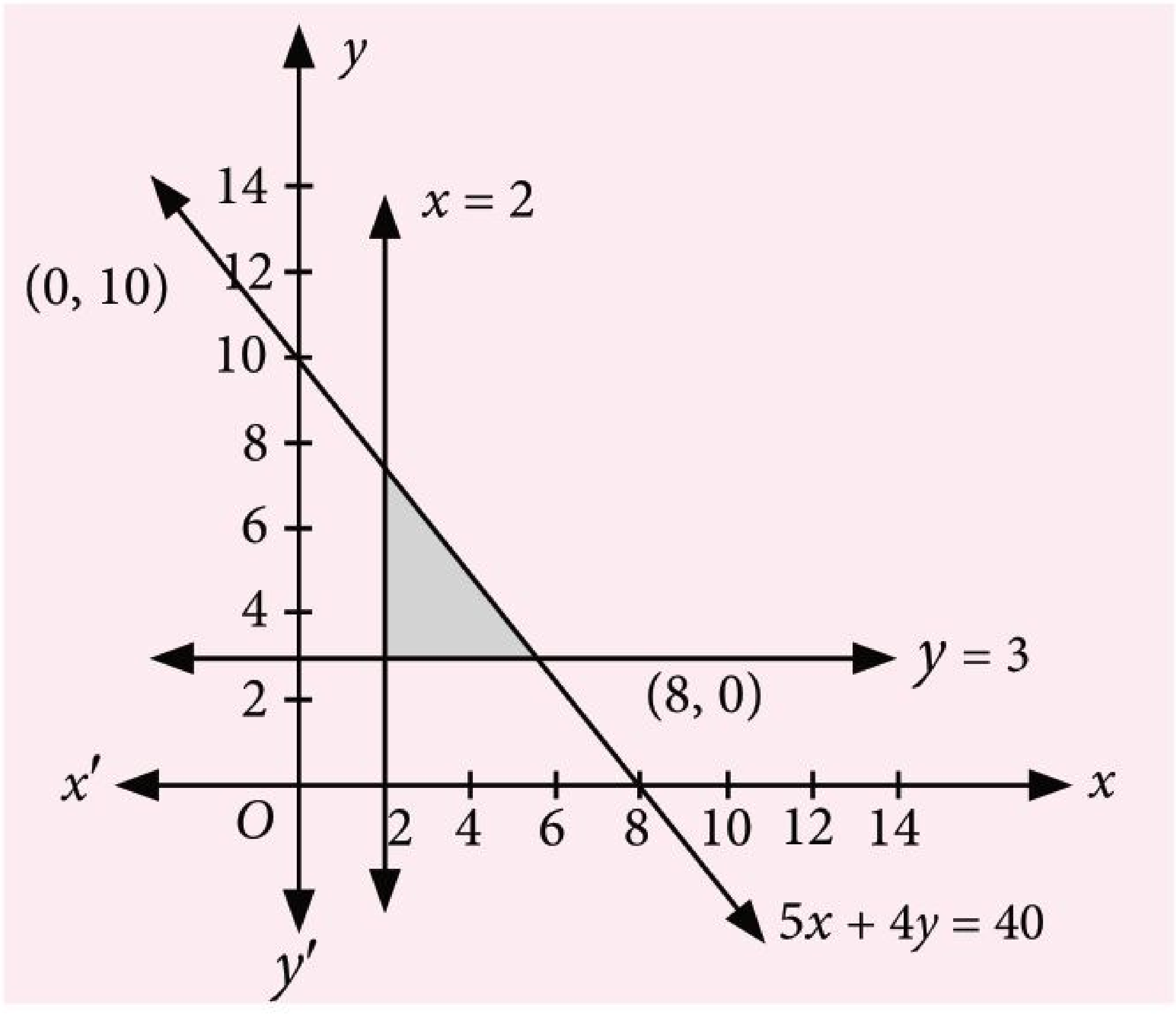
Converting the given inequalities into equalities, we get

$5x + 4y = 40$ ...(i)

$x = 2$ ...(ii)

$y = 3$ ...(iii)

Now, draw the graphs of (i), (ii) and (iii) as shown below.



So, the shaded region including all the points on the lines represents the solution set of the given system of inequalities.

10. Let $y = \cos(5x + 2)$

Then, using first principle

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(5(x+h)+2) - \cos(5x+2)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(5x+5h+2) - \cos(5x+2)}{h}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(5x + \frac{5h}{2} + 2\right)\sin\left(\frac{5h}{2}\right)}{h}$$

$$= \lim_{h\to 0} -5\sin\left(5x + \frac{5h}{2} + 2\right)\frac{\sin\left(\frac{5h}{2}\right)}{\frac{5h}{2}}$$

$= -5\sin(5x + 2)$

11. When two dice are thrown, there are $6 \times 6 = 36$ possible outcomes.

$\therefore \; S = \{$(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$\therefore$ A = The sum is greater than 8

= {(3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)}

B = 2 occurs on either die

= {(1, 2), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (4, 2), (5, 2), (6, 2)}

C = The sum is at least 7 and a multiple of 3

= {(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)}

Also, $A \cap B = \phi$, $B \cap C = \phi$

and $A \cap C$ = {(3, 6), (4, 5), (5, 4), (6, 3), (6, 6)}

(i) Since $A \cap B = \phi$. So A and B are mutually exclusive.

(ii) Since $B \cap C = \phi$. So B and C are mutually exclusive.

(iii) Since $A \cap C \neq \phi$. So A and C are not mutually exclusive.

OR

Let S be the sample space associated with the experiment of throwing a pair of dice.

Then, $n(S) = 36$.

$\therefore$ Total number of possible outcomes = 36

Consider the following events.

A : The sum of the numbers on two faces is divisible by 3

B : The sum of the numbers on two faces is divisible by 4.

Then, A = {(1, 2), (2, 1), (1, 5), (5, 1), (3, 3), (2, 4), (4, 2), (3, 6), (6, 3), (4, 5), (5, 4), (6, 6)}

B = {(2, 2), (1, 3), (3, 1), (2, 6), (6, 2), (4, 4), (3, 5), (5, 3), (6, 6)} and $A \cap B$ ={(6, 6)}

$$\therefore\ P(A)=\frac{12}{36}=\frac{1}{3},\ P(B)=\frac{9}{36}=\frac{1}{4} \text{ and } P(A\cap B)=\frac{1}{36}$$

(i) Required probability = $P(\overline{A}\cap\overline{B}) = P(\overline{A\cup B})$

$= 1 - P(A \cup B) = 1 - \{P(A) + P(B) - P(A \cap B)\}$

$$=1-\left\{\frac{1}{3}+\frac{1}{4}-\frac{1}{36}\right\}=\frac{4}{9}$$

(ii) Required probability = $P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B) = \frac{1}{3}+\frac{1}{4}-\frac{1}{36}=\frac{5}{9}.$$

12. L.H.S. $= \sin^3 x+\sin^3\left(\frac{2\pi}{3}+x\right)+\sin^3\left(\frac{4\pi}{3}+x\right)$

$$=\frac{3\sin x-\sin 3x}{4}+\frac{3\sin\left(\frac{2\pi}{3}+x\right)-\sin(2\pi+3x)}{4}+\frac{3\sin\left(\frac{4\pi}{3}+x\right)-\sin(4\pi+3x)}{4}$$

$$=\frac{1}{4}\left[3\left\{\sin x+\sin\left(\frac{2\pi}{3}+x\right)+\sin\left(\frac{4\pi}{3}+x\right)\right\}-\{\sin 3x+\sin 3x+\sin 3x\}\right]$$

$[\because \sin(2\pi+x)=\sin x \text{ and } \sin(4\pi+x)=\sin x]$

$$=\frac{1}{4}\left[3\left\{\sin x+2\cdot\sin(\pi+x)\cos\left(\frac{\pi}{3}\right)\right\}-3\sin 3x\right]$$

$$=\frac{1}{4}\left[3\left\{\sin x-2\cdot\sin x.\left(\frac{1}{2}\right)\right\}-3\sin 3x\right]$$

$[\therefore \sin(180° + x) = -\sin x]$

$$=\frac{1}{4}[3\{\sin x-\sin x\}-3\sin 3x]=-\frac{3}{4}\sin 3x=\text{R.H.S.}$$

13. Let S and S' be two foci of the required ellipse. Then, the coordinates of S and S' are (2, 3) and (–2, 3) respectively.

$\therefore$ $SS' = 4$

Let $2a$ and $2b$ be the lengths of the axes of the ellipse and e be its eccentricity.

Then, $SS' = 2ae \Rightarrow 2ae = 4 \Rightarrow ae = 2$.

Now, $b^2 = a^2(1 - e^2) \Rightarrow 5 = a^2 - 2^2 \Rightarrow a = 3$.

Let $P(x, y)$ be any point on the ellipse. Then,

$SP + S'P = 2a$

$$\Rightarrow \sqrt{(x-2)^2+(y-3)^2}+\sqrt{(x+2)^2+(y-3)^2}=6$$

$$\Rightarrow [(x-2)^2+(y-3)^2]-[(x+2)^2+(y-3)^2]$$

$$=36-12\left\{\sqrt{(x+2)^2+(y-3)^2}\right\}$$

$$\Rightarrow -8x=36-12\left\{\sqrt{(x+2)^2+(y-3)^2}\right\}$$

$$\Rightarrow (2x+9)^2=9\{(x+2)^2+(y-3)^2\}$$

$\Rightarrow$ $5x^2 + 9y^2 - 54y + 36 = 0$ is the required equation of the ellipse.

14. (i) Now, 2 History books can be chosen in ${}^5P_2$ ways, 1 Maths book can be chosen in ${}^3P_1$ ways and 1 Science book can be chosen in ${}^4P_1$ ways.

$\therefore$ Required number of ways = ${}^5P_2 \times {}^3P_1 \times {}^4P_1 = 240$.

(ii) Number of ways of arranging History books = 5!

Number of ways of arranging Maths books = 3!

Number of ways of arranging Science books = 4!

$\therefore$ Required number of ways if the books of same subject are put together = 3!· 5!· 3! · 4!.

MONTHLY TEST
DRIVE
XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 9 : Conic Sections

Time Taken : 60 Min.

Only One Option Correct Type

1. Equation of the normal at a point on the parabola $y^2 = 36x$, whose ordinate is three times its abscissa is
(a) $2x + 3y + 44 = 0$ (b) $2x - 3y + 44 = 0$
(c) $2x + 3y - 44 = 0$ (d) $2x - 3y = 0$

2. Equation of hyperbola passing through origin and whose asymptotes are $3x + 4y = 5$ and $4x + 3y = 5$, is:
(a) $x^2 - y^2 = 1$
(b) $12x^2 + 12y^2 + 35xy - 15x - 15y = 0$
(c) $12x^2 + 12y^2 + 25xy - 35x - 35y = 0$
(d) $12x^2 + 12y^2 + 25xy - 25x - 25y = 0$

3. The circle with equation $x^2 + y^2 = 1$ intersects the line $y = 7x + 5$ at two distinct point A and B. Let C be the point at which the positive x-axis intersects the circle. The angle ACB is:
(a) $\tan^{-1}\left(\frac{4}{3}\right)$ (b) $\tan^{-1}\left(\frac{3}{4}\right)$
(c) $\tan^{-1}(1)$ (d) $\tan^{-1}\left(\frac{3}{2}\right)$

4. Tangents drawn to parabola $y^2 = 4ax$ at the points A and B intersect at C. If 'S' be the focus of the parabola, then SA, SC and SB are in:
(a) A.P. (b) G.P. (c) H.P. (d) A.G.P

5. If a tangent of slope 'm' at a point of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through $(2a, 0)$ and if 'e' denotes the eccentricity of the ellipse, then:
(a) $m^2 + e^2 = 1$ (b) $2m^2 + e^2 = 1$
(c) $3m^2 + e^2 = 1$ (d) None of these

6. If the tangent and the normal to a rectangular hyperbola $xy = c^2$ at a point cut off intercepts a_1, a_2 on one axis and b_1, b_2 the other axis, then $a_1a_2 + b_1b_2$ is equal to
(a) -1 (b) 0 (c) 1 (d) $\sqrt{2}$

One or More Than One Option(s) Correct Type

7. If the conics whose equations are

$$S_1 : (\sin^2\theta)x^2 + (2h\tan\theta)\,xy + (\cos^2\theta)\,y^2 + 32x + 16y + 19 = 0$$

$$S_2 : (\cos^2\theta)\,x^2 - (2h'\cot\theta)\,xy + (\sin^2\theta)y^2 + 16x + 32y + 19 = 0$$

intersect in four concyclic points, where $\theta \in \left[0, \frac{\pi}{2}\right]$, then the correct statements (s) can be :
(a) $h + h' = 0$ (b) $h - h' = 0$
(c) $\theta = \pi/4$ (d) None of these

8. Tangent is drawn at any point (x_1, y_1) other than the vertex on the parabola $y^2 = 4ax$. if tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of contact pass through a fixed point (x_2, y_2), then
(a) x_1, a, x_2 and in GP (b) $\frac{y_1}{2}, a, y_2$ are in GP
(c) $-4, \frac{y_1}{y_2}, \frac{x_1}{x_2}$ are in GP (d) $x_1x_2 + y_1y_2 = a^2$

9. If the tangent to the ellipse $x^2 + 4y^2 = 16$ at the point $P(\theta)$ is a normal to the circle $x^2 + y^2 - 8x - 4y = 0$, then θ can be equal to :
(a) $\pi/3$ (b) $\pi/2$ (c) 0 (d) $-\pi/4$

10. For the hyperbola $9x^2 - 16y^2 - 18x + 32y - 151 = 0$,
(a) one of the directrix is $x = 21/5$
(b) the length of latus rectus is $9/2$
(c) foci are $(6, 1)$ and $(-4, 1)$
(d) the eccentricity is $5/4$

11. The locus of the image of the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1, (a > b)$ with respect to any of the tangents to the ellipse is

(a) $(x+4)^2+y^2=100$ (b) $(x+2)^2+y^2=50$
(c) $(x-4)^2+y^2=100$ (d) $(x-2)^2+y^2=50$

12. Point M moved on the circle $(x-4)^2+(y-8)^2=20$. Then it broke away from it and moving along a tangent to the circle cut the x-axis at point $(-2, 0)$. The coordinates of the point on the circle at which the moving point broke away is

(a) $\left(\frac{42}{5}, \frac{36}{5}\right)$ (b) $\left(-\frac{2}{5}, \frac{44}{5}\right)$
(c) $(6, 4)$ (d) $(2, 4)$

Comprehension Type

Paragraph for Q. No. 13 to 15

For $P: y^2 = 8x,\ E: \frac{x^2}{4}+\frac{y^2}{15}=1$

13. Equation of a tangent common to both the parabola P and the ellipse E is
(a) $x \pm 2y + 8 = 0$ (b) $x + 2y \mp 8 = 0$
(c) $x + 2y - 8 = 0$ (d) $x - 2y - 8 = 0$

14. Equation of the normal at the point of contact of the common tangent, which makes on acute angle with the positive direction of x-axis, to the parabola P is
(a) $2x + y = 24$ (b) $2x + y + 24 = 0$
(c) $2x + y = 48$ (d) $2x + y + 48 = 0$

15. Points of contact of a common tangent to P and E on the ellipse is

(a) $\left(\frac{1}{2}, \frac{15}{4}\right)$ (b) $\left(-\frac{1}{2}, \frac{15}{4}\right)$
(c) $\left(\frac{1}{2}, -\frac{15}{4}\right)$ (d) $\left(-\frac{1}{2}, -\frac{13}{4}\right)$

Matrix Match Type

16. Match the following :

	Column-I	Column-II	
P.	If the mid-point of a chord of the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ is (0, 3), then length of the chord is $4k/5$ then k is	1.	6
Q.	If the line $y = x + \lambda$ touches the ellipse $9x^2 + 16y^2 = 144$, then the sum of values of λ is	2.	8
R.	If the distance between a focus and corresponding directrix of an ellipse be 8 and the eccentricity be 1/2, then length of the minor axis is $k/\sqrt{3}$, then k is	3.	0
S.	Sum of distances of a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ from the foci	4.	16
T.		5.	7

	P	Q	R	S
(a)	2	3	4	1
(b)	3	2	4	2
(c)	1	3	4	5
(d)	3	2	4	1

Numerical Answer Type

17. If angle between tangents drawn to $x^2 + y^2 - 12x - 16y = 0$ at points where curve is cut by $5y = 5x + c\,(c > 0)$ is $\pi/2$ and $c = 10k$, then k equals.

18. Normals are drawn from the point P on a parabola $y^2 = 4x$ with slopes $m_1 \cdot m_2 = \alpha$ is a part of the parabola itself, then the value of α is :

19. If the tangent at any point of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ makes an angle α with the major axis and an angle β with the focal radius of the point of contact, then the eccentricity of the ellipse is given by $e=\left(\frac{\cos\beta}{\cos\alpha}\right)\frac{\lambda}{5}$. Find λ.

20. Let C is the centre of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$. if the tangent at any point P on the hyperbola meets the straight lines $3x - 4y = 0$ and $3x + 4y = 0$ in the points Q and R respectively, then $\frac{CQ \cdot CR}{5} =$

◆◆

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ……
No. of questions correct ……
Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

CBSE warm-up!

Term-II

CLASS-XII

Practice Paper 2022

Time allowed : 2 Hours
Maximum marks : 40

General Instructions :

1. *This question paper contains three sections - A, B and C. Each part is compulsory.*
2. *Section - A has 6 short answer type (SA1) questions of 2 marks each.*
3. *Section - B has 4 short answer type (SA2) questions of 3 marks each.*
4. *Section - C has 4 long answer type questions (LA) of 4 marks each.*
5. *There is an internal choice in some of the questions.*
6. *Q14 is a case-based problem having 2 sub parts of 2 marks each.*

SECTION - A

1. A machine produces parts that are either good (90%), slightly defective (2%), or obviously defective (8%). Produced parts get passed through an automatic inspection machine, which is able to detect any part that is obviously defective and discard it. What is the probability of the parts that make it through the inspection machine and get shipped?

2. Solve the following differential equation :

$$\frac{dy}{dx}+y=\cos x-\sin x$$

3. Write the number of vectors of unit length perpendicular to both the vectors $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$.

4. Evaluate : $\int \sin^{-1}(2x)dx$

OR

Evaluate : $\int_1^2 \left[\frac{1}{x}-\frac{1}{2x^2}\right]e^{2x}dx$

5. Find the vector and cartesian equations of the line which passes through the point (5, 2, –4) and is parallel to the vector $3\hat{i}+2\hat{j}-8\hat{k}$.

6. A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
$P(X)$	a	$4a$	$3a$	$7a$	$8a$	$10a$	$6a$	$9a$

Find $P(X < 3)$, $P(X \geq 4)$, $P(0 < X < 5)$ respectively.

SECTION - B

7. Solve the following differential equation

$$y^2dx + (x^2 - xy + y^2)dy = 0$$

OR

Solve the differential equation

$$x\log x\frac{dy}{dx}+y=\frac{2}{x}\log x.$$

8. Evaluate : $\int \frac{x^2+x+1}{(x^2+1)(x+2)}dx$

9. If $\hat{i}+\hat{j}+\hat{k},\ 2\hat{i}+5\hat{j},\ 3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear or not.

OR

If $\vec{a}=2\hat{i}-3\hat{j}+\hat{k},\vec{b}=-\hat{i}+\hat{k}$, $\vec{c}=2\hat{j}-\hat{k}$ are three vectors, find the area of the parallelogram having diagonals $(\vec{a}+\vec{b})$ and $(\vec{b}+\vec{c})$.

10. Find the coordinates of a point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ which is at a distance of $\frac{6}{\sqrt{2}}$ from the point (1, 2, 3).

SECTION - C

11. Evaluate : $\int \frac{x^2}{(x\sin x+\cos x)^2}dx$

12. Given that, the events A and B are such that $P(A)=\frac{1}{2}$, $P(A\cup B)=\frac{3}{5}$ and $P(B) = p$. Then, find the probabilities of B if A and B are (i) mutually exclusive (ii) independent.

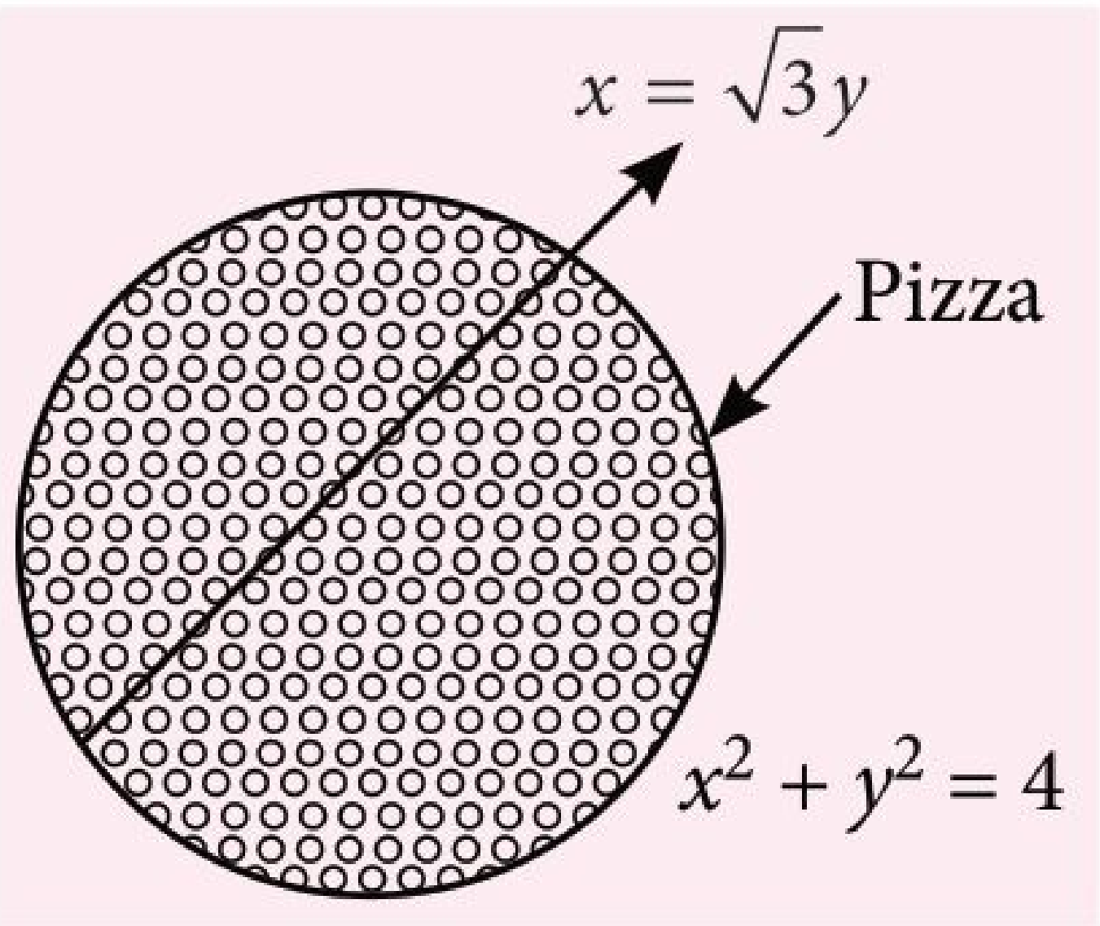
OR

Two cards from an ordinary deck of 52 cards are missing. What is the probability that a random card drawn from this deck is a spade?

13. If lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then find the value of k and hence find the equation of the plane containing these lines.

CASE-BASED / DATA-BASED

14. A child cuts a pizza with a knife. Pizza is circular in shape which is represented by $x^2 + y^2 = 4$ and sharp edge of knife represents a straight line given by $x=\sqrt{3}y$.



Based on the above information, answer the following questions.

(i) Find the point(s) of intersection of the edge of knife (line) and pizza shown in the figure.

(ii) What is the area of the region bounded by circular pizza and edge of knife in first quadrant?

SOLUTIONS

1. Let G, SD, OD be the events that a randomly chosen part is good, slightly defective, obviously defective respectively.

Then, $P(G) = 0.90$, $P(SD) = 0.02$, and $P(OD) = 0.08$

Required probability = $P(G \mid OD^c)$

$$=\frac{P(G\cap OD^c)}{P(OD^c)}=\frac{P(G)}{1-P(OD)}=\frac{0.90}{1-0.08}=\frac{90}{92}=0.978$$

2. We have, $\frac{dy}{dx}+y=\cos x-\sin x$, which is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$, where $P=1$, $Q=\cos x-\sin x$

$\therefore$ I.F. $=e^{\int dx}=e^x$

The solution of given differential equation is

$ye^x=\int e^x(\cos x-\sin x)dx+C$

$\Rightarrow\ ye^x=e^x\cos x+C\Rightarrow y=\cos x+Ce^{-x}$

3. Given, $\vec{a}=2\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=\hat{j}+\hat{k}$

Unit vectors perpendicular to $\vec{a}$ and $\vec{b}$ are

$\pm\left(\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}\right)$.

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & 2\\ 0 & 1 & 1\end{vmatrix}=-\hat{i}-2\hat{j}+2\hat{k}$

$\therefore$ Unit vectors perpendicular to $\vec{a}$ and $\vec{b}$ are

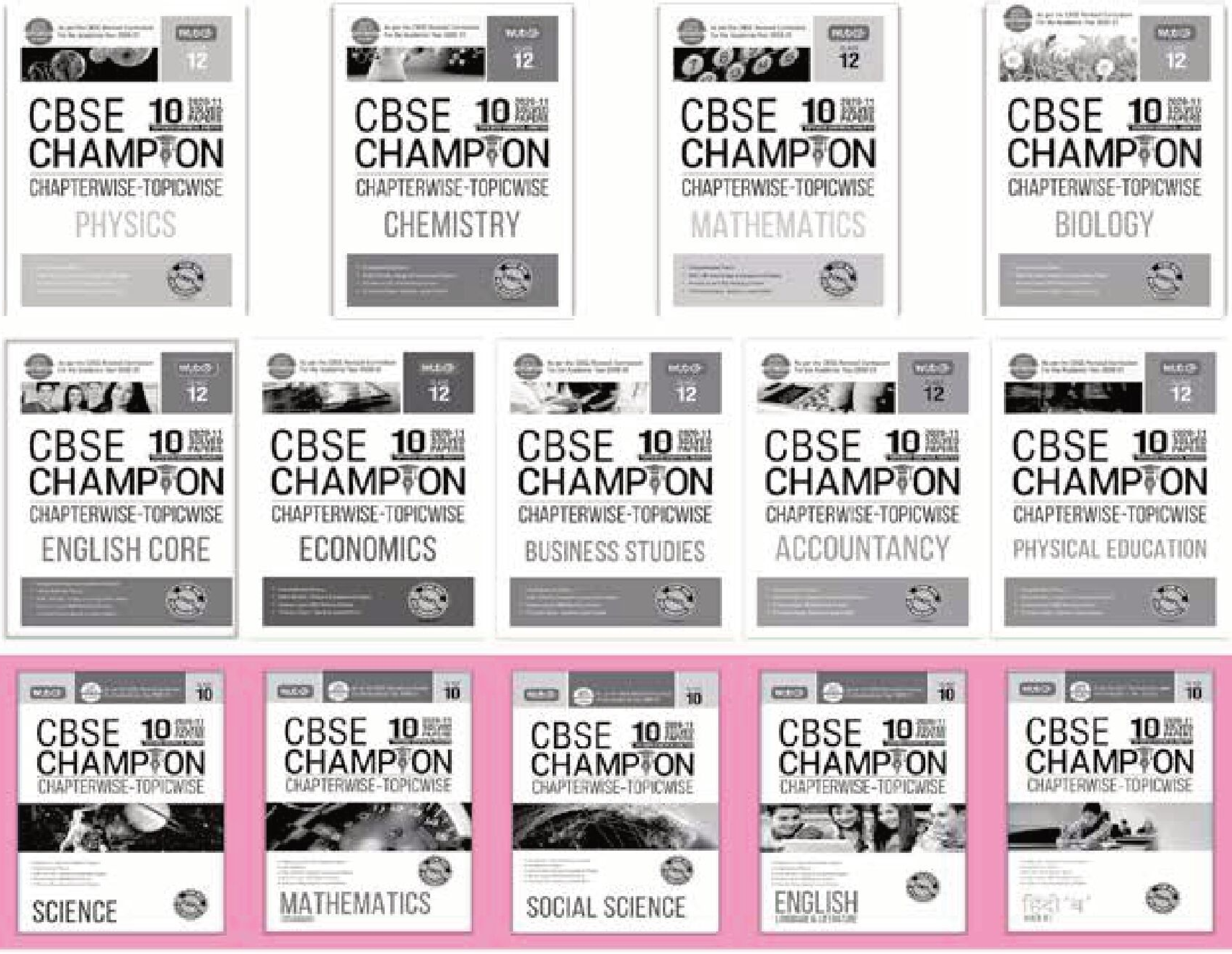
$$\pm\frac{(-\hat{i}-2\hat{j}+2\hat{k})}{\sqrt{(-1)^2+(-2)^2+(2)^2}}=\pm\left(-\frac{1}{3}\hat{i}-\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}\right)$$

So, there are two unit vectors perpendicular to the given vectors.

4. Let $I=\int\sin^{-1}(2x)\,dx=\int 1\cdot\sin^{-1}(2x)\,dx$

$$=\sin^{-1}(2x)x-\int\left(\frac{1}{\sqrt{1-4x^2}}\frac{d}{dx}(2x)\cdot x\right)dx$$

CBSE CHAMPION Chapterwise -Topicwise Solved Papers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$$= x\sin^{-1}(2x) - \int \frac{2x}{\sqrt{1-4x^2}}\,dx \;= x\sin^{-1}(2x) + \int \frac{dt}{4\sqrt{t}}$$

(Putting $1-4x^2 = t \Rightarrow -8x\,dx = dt$)

$$= x\sin^{-1}(2x) + \frac{2}{4}(t)^{1/2} + C$$

$$= x\sin^{-1}(2x) + \frac{1}{2}\sqrt{1-4x^2} + C$$

OR

Let $I = \int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2}\right] e^{2x}\,dx$

Putting $2x = y \Rightarrow 2dx = dy$

As $x \to 1 \Rightarrow y \to 2$ and $x \to 2 \Rightarrow y \to 4$

$$\therefore \quad I = \frac{1}{2}\int_2^4 \left[\frac{2}{y} - \frac{2}{y^2}\right] e^y\,dy = \int_2^4 \left[\frac{1}{y} - \frac{1}{y^2}\right] e^y\,dy$$

$$= \left[e^y \cdot \frac{1}{y}\right]_2^4 = \frac{1}{4}e^4 - \frac{1}{2}e^2 = \frac{e^2}{2}\left(\frac{e^2}{2} - 1\right)$$

5. We have, $\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - 8\hat{k}$

Therefore, the vector equation of the line is

$\vec{r} = (5\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$

Now, as $\vec{r}$ is the position vector of any point $P(x, y, z)$ on the line.

Therefore, $x\hat{i} + y\hat{j} + z\hat{k} = 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$

$\Rightarrow \quad x\hat{i} + y\hat{j} + z\hat{k} = (5 + 3\lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (-4 - 8\lambda)\hat{k}$

Eliminating λ, we get $\frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-8}$,

which is the equation of the line in cartesian form.

6. We know that, $\Sigma p_i = 1$

$\therefore \quad a + 4a + 3a + 7a + 8a + 10a + 6a + 9a = 1$

$\Rightarrow \quad 48a = 1 \Rightarrow \quad a = \frac{1}{48}$

Now, $P(X < 3) = P(0) + P(1) + P(2)$

$= a + 4a + 3a = 8a = 8 \cdot \frac{1}{48} = \frac{1}{6}$

$P(X \geq 4) = P(4) + P(5) + P(6) + P(7)$

$= 8a + 10a + 6a + 9a = 33a = \frac{33}{48}$

and $P(0 < X < 5) = P(1) + P(2) + P(3) + P(4)$

$= 4a + 3a + 7a + 8a = 22a = \frac{22}{48} = \frac{11}{24}$

7. We have, $y^2\,dx + (x^2 - xy + y^2)\,dy = 0$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y^2}{x^2 - xy + y^2}$$

This is homogeneous differential equation.

$\therefore \quad$ Put $y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$$v + x\frac{dv}{dx} = \frac{-v^2x^2}{x^2 - vx^2 + v^2x^2} \quad \Rightarrow \quad v + x\frac{dv}{dx} = \frac{-v^2}{1 - v + v^2}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{-v^2}{1 - v + v^2} - v \quad \Rightarrow x\frac{dv}{dx} = \frac{-v - v^3}{1 - v + v^2}$$

$$\Rightarrow \frac{1 - v + v^2}{v(1 + v^2)}\,dv = -\frac{1}{x}\,dx$$

Integrating both sides, we get

$$\int \frac{1 + v^2}{v(1 + v^2)}\,dv - \int \frac{v}{v(1 + v^2)}\,dv = -\int \frac{1}{x}\,dx$$

$$\Rightarrow \quad \int \frac{1}{v}\,dv - \int \frac{1}{1 + v^2}\,dv = -\int \frac{1}{x}\,dx$$

$\Rightarrow \quad \log|v| - \tan^{-1}v = -\log|x| + \log C$

$\Rightarrow \quad \log\left|\frac{vx}{C}\right| = \tan^{-1}v \qquad \Rightarrow \quad \left|\frac{vx}{C}\right| = e^{\tan^{-1}v}$

$\Rightarrow \quad |y| = Ce^{\tan^{-1}y/x}$ is the required solution.

OR

We have, $x\log x\frac{dy}{dx} + y = \frac{2}{x}\log x$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x^2}$$

It is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{x\log x}, Q = \frac{2}{x^2}$

$$\therefore \quad \text{I.F.} = e^{\int \frac{1}{x\log x}dx} = e^{\log\log x} = \log x$$

$$\therefore \quad y(\log x) = \int (\log x)\frac{2}{x^2}\,dx$$

$$\Rightarrow y(\log x) = \log x\int \frac{2}{x^2}\,dx - \int\left(\frac{d}{dx}(\log x)\int \frac{2}{x^2}\,dx\right)dx$$

$$\Rightarrow \ y(\log x) = \log x\left(-\frac{2}{x}\right) + \int \frac{2}{x^2}\,dx$$

$$\Rightarrow \ y(\log x) = \log x\left(-\frac{2}{x}\right) - \frac{2}{x} + \text{C}$$

8. Let $I = \int \frac{x^2 + x + 1}{(x^2 + 1)(x + 2)}\,dx$ ...(1)

Let $\frac{x^2 + x + 1}{(x^2 + 1)(x + 2)} = \frac{Ax + B}{x^2 + 1} + \frac{C}{x + 2}$ ...(2)

$\Rightarrow \quad x^2 + x + 1 = (Ax + B)(x + 2) + C(x^2 + 1)$

Put $x = 0, 1, -2$ in it to get

$1 = 2B + C$; $3 = 3(A + B) + 2C$ and $3 = 5C$

$$\Rightarrow \quad C = \frac{3}{5}, B = \frac{1}{5} \text{ and } A = \frac{2}{5}$$

Hence, from (2), we have

$$\frac{x^2+x+1}{(x^2+1)(x+2)} = \frac{\left(\frac{2}{5}x+\frac{1}{5}\right)}{x^2+1} + \frac{\frac{3}{5}}{x+2}$$

$$= \frac{1}{5} \cdot \frac{2x+1}{x^2+1} + \frac{3}{5} \cdot \frac{1}{x+2}$$

$$\therefore \quad I = \frac{1}{5}\int \frac{2x+1}{x^2+1}dx + \frac{3}{5}\int \frac{1}{x+2}dx$$

$$= \frac{1}{5}\left[\int \frac{2x}{x^2+1}dx + \int \frac{dx}{x^2+1}\right] + \frac{3}{5}\int \frac{dx}{x+2}$$

$$= \frac{1}{5}\left[\log|x^2+1| + \tan^{-1}x\right] + \frac{3}{5}\log|x+2| + C_1$$

9. Given, position vector of $A = \hat{i}+\hat{j}+\hat{k}$

Position vector of $B = 2\hat{i}+5\hat{j}$

Position vector of $C = 3\hat{i}+2\hat{j}-3\hat{k}$

Position vector of $D = \hat{i}-6\hat{j}-\hat{k}$

$\therefore \quad \overrightarrow{AB} = (2\hat{i}+5\hat{j}) - (\hat{i}+\hat{j}+\hat{k}) = \hat{i}+4\hat{j}-\hat{k}$ and

$\overrightarrow{CD} = (\hat{i}-6\hat{j}-\hat{k}) - (3\hat{i}+2\hat{j}-3\hat{k}) = -2\hat{i}-8\hat{j}+2\hat{k}$

Now $|\overrightarrow{AB}| = \sqrt{(1)^2+(4)^2+(-1)^2} = \sqrt{18}$

$|\overrightarrow{CD}| = \sqrt{(-2)^2+(-8)^2+(2)^2} = \sqrt{72} = 2\sqrt{18}$

Let θ be the angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

$$\therefore \cos\theta = \frac{\overrightarrow{AB}\cdot\overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|} = \frac{(\hat{i}+4\hat{j}-\hat{k})\cdot(-2\hat{i}-8\hat{j}+2\hat{k})}{(\sqrt{18})(2\sqrt{18})}$$

$$= \frac{-2-32-2}{36} = \frac{-36}{36} = -1$$

$\Rightarrow \quad \cos\theta = -1 \quad \Rightarrow \quad \theta = \pi$

Since, angle between $\overrightarrow{AB}$ and $\overrightarrow{CD}$ is 180°.

$\therefore \quad \overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear.

OR

Here, $\vec{a} = 2\hat{i}-3\hat{j}+\hat{k}$; $\vec{b} = -\hat{i}+\hat{k}$; $\vec{c} = 2\hat{j}-\hat{k}$

$\therefore \quad \vec{a}+\vec{b} = (2\hat{i}-3\hat{j}+\hat{k}) + (-\hat{i}+\hat{k}) = \hat{i}-3\hat{j}+2\hat{k}$,

$\vec{b}+\vec{c} = (-\hat{i}+\hat{k}) + (2\hat{j}-\hat{k}) = -\hat{i}+2\hat{j}$

$$\therefore (\vec{a}+\vec{b})\times(\vec{b}+\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ -1 & 2 & 0 \end{vmatrix} = -4\hat{i}-2\hat{j}-\hat{k}$$

$\therefore$ Area of a parallelogram whose diagonals are $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ $= \frac{1}{2}\left|(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})\right| = \frac{1}{2}\left|-4\hat{i}-2\hat{j}-\hat{k}\right|$

$$= \frac{1}{2}\sqrt{(-4)^2+(-2)^2+(-1)^2} = \frac{\sqrt{21}}{2} \text{ sq.units.}$$

10. Given equation of line is

$$\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda \text{ (say)}$$

$\Rightarrow \quad x = 3\lambda - 2, y = 2\lambda - 1, z = 2\lambda + 3$

$\therefore$ Coordinates of any point on the line are

$(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$ ...(i)

The distance between this point and (1, 2, 3) is $\frac{6}{\sqrt{2}}$

$$\therefore \quad \sqrt{(3\lambda-2-1)^2+(2\lambda-1-2)^2+(2\lambda+3-3)^2} = \frac{6}{\sqrt{2}}$$

$$\Rightarrow (3\lambda-3)^2+(2\lambda-3)^2+(2\lambda)^2 = \frac{36}{2}$$

$\Rightarrow \quad 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 9 - 12\lambda + 4\lambda^2 = 18$

$\Rightarrow \quad 17\lambda^2 - 30\lambda = 0 \Rightarrow \lambda(17\lambda-30) = 0 \Rightarrow \lambda = 0, \frac{30}{17}$

Substituting the values of λ in (i), we get the required point is $(-2, -1, 3)$ or $\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$.

11. Let $I = \int \frac{x^2}{(x\sin x+\cos x)^2}dx$

$$= \int \frac{x\cos x}{(x\sin x+\cos x)^2}\cdot x\sec x\,dx \quad \text{...(1)}$$

Let $I_1 = \int \frac{x\cos x}{(x\sin x+\cos x)^2}dx$

Put $x\sin x + \cos x = t$

$\Rightarrow \quad (x\cos x + 1\cdot\sin x - \sin x)dx = dt$

$\Rightarrow \quad x\cos x\,dx = dt$

$$\therefore \quad I_1 = \int \frac{dt}{t^2} = -\frac{1}{t} = \frac{-1}{x\sin x+\cos x}$$

Now integrating (1) by parts taking $x\sec x$ as the first function and using I_1, we get

$$I = x\sec x\cdot\frac{-1}{x\sin x+\cos x}$$
$$-\int(1\cdot\sec x + x\sec x\tan x)\cdot\frac{-dx}{x\sin x+\cos x}$$

$$=\frac{-x\sec x}{x\sin x+\cos x}+\int \sec x\left(1+\frac{x\sin x}{\cos x}\right)\frac{dx}{x\sin x+\cos x}$$

$$=\frac{-x\sec x}{x\sin x+\cos x}+\int \sec^2 x\,dx$$

$$=\frac{-x\sec x}{x\sin x+\cos x}+\tan x+C$$

12. (i) When A and B are mutually exclusive, then

$A \cap B = \phi \Rightarrow P(A \cap B) = 0$

$\because \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\therefore \quad \frac{3}{5}=\frac{1}{2}+p-0 \Rightarrow \quad p=\frac{3}{5}-\frac{1}{2}=\frac{6-5}{10}=\frac{1}{10}$$

(ii) When A and B are independent events, then

$$P(A\cap B)=P(A)\cdot P(B)=\frac{1}{2}p \quad \left[\because P(A)=\frac{1}{2}\text{ and } P(B)=p\right]$$

Now, $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$$\Rightarrow \quad \frac{3}{5}=\frac{1}{2}+p-\frac{1}{2}p \Rightarrow \quad \frac{3}{5}-\frac{1}{2}=\frac{2p-p}{2}$$

$$\Rightarrow \quad \frac{p}{2}=\frac{6-5}{10} \quad \Rightarrow \quad p=\frac{2}{10}=\frac{1}{5}$$

OR

Let E be the event that the randomly drawn card is a spade. Let F_i be the event that i spades are missing from the 52-card, for $i = 0, 1, 2$.

We want $P(E)$, which we compute by conditioning on how many spades are missing from the original deck :

$P(E) = P(E|F_0)\, P(F_0) + P(E|F_1)\, P(F_1) + P(E|F_2)\, P(F_2)$

$$=\frac{13}{50}\frac{\binom{13}{0}\binom{39}{2}}{\binom{52}{2}}+\frac{12}{50}\frac{\binom{13}{1}\binom{39}{1}}{\binom{52}{2}}+\frac{11}{50}\frac{\binom{13}{2}\binom{39}{0}}{\binom{52}{2}}=\frac{1}{4}$$

13. We have, $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}=\lambda\text{ (say)}$

$\Rightarrow \quad x = 2\lambda + 1,\ y = 3\lambda - 1,\ z = 4\lambda + 1$

Since, given both lines intersect.

$\therefore$ Point $(2\lambda + 1, 3\lambda - 1, 4\lambda + 1)$ satisfies

$$\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$$

$$\Rightarrow \quad \frac{2\lambda+1-3}{1}=\frac{3\lambda-1-k}{2}=\frac{4\lambda+1}{1} \qquad \text{...(i)}$$

$$\Rightarrow \quad 2\lambda - 2 = 4\lambda + 1 \Rightarrow 2\lambda = -3 \Rightarrow \lambda=\frac{-3}{2}$$

Putting this value of λ in (i), we get

$$2\left(\frac{-3}{2}\right)-2=\frac{3\left(\frac{-3}{2}\right)-1-k}{2}$$

$$\Rightarrow \quad -5\times 2=-\frac{11}{2}-k \Rightarrow \quad k=\frac{-11}{2}+10=\frac{9}{2}$$

Now, required equation of plane is

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix}=0$$

$\Rightarrow \quad -5x + 5 + 2y + 2 + z - 1 = 0$

$\Rightarrow \quad 5x - 2y - z - 6 = 0$

14. **(i)** We have, $x^2 + y^2 = 4$ ...(i)

and $x=\sqrt{3}y$ ...(ii)

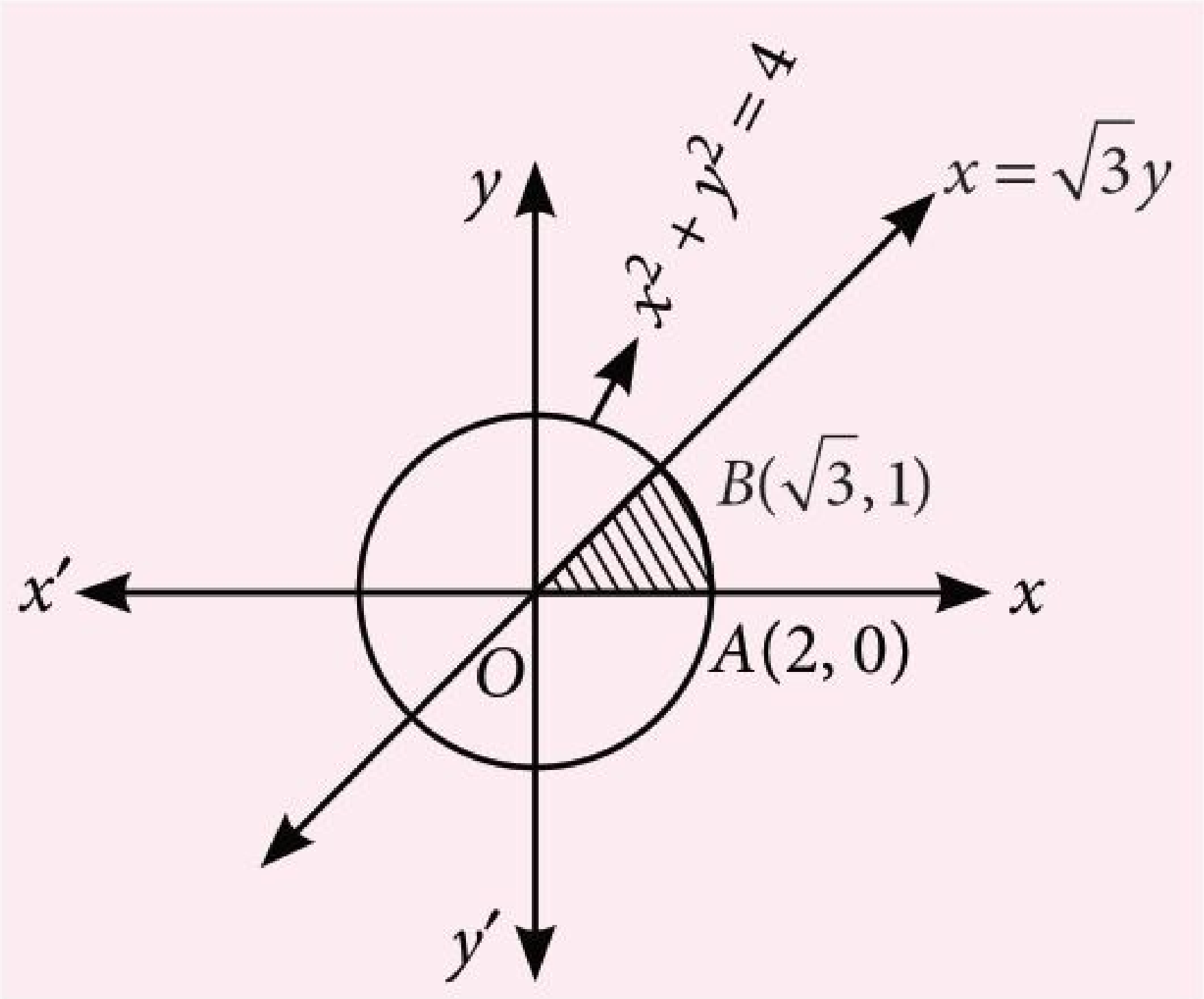
From (i) and (ii), we get

$3y^2 + y^2 = 4 \Rightarrow 4y^2 = 4 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$

From (ii), $x=\sqrt{3},-\sqrt{3}$

$\therefore$ Points of intersection of pizza and edge of knife are $(\sqrt{3},1),(-\sqrt{3},-1)$.

(ii)



$$\text{Required area} = \int_0^{\sqrt{3}}\frac{x}{\sqrt{3}}dx+\int_{\sqrt{3}}^{2}\sqrt{4-x^2}\,dx$$

$$=\frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{\sqrt{3}}+\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{\sqrt{3}}^{2}$$

$$=\frac{1}{\sqrt{3}}\left[\frac{3}{2}-0\right]+\left[2\sin^{-1}(1)-\left(\frac{\sqrt{3}}{2}+2\sin^{-1}\frac{\sqrt{3}}{2}\right)\right]$$

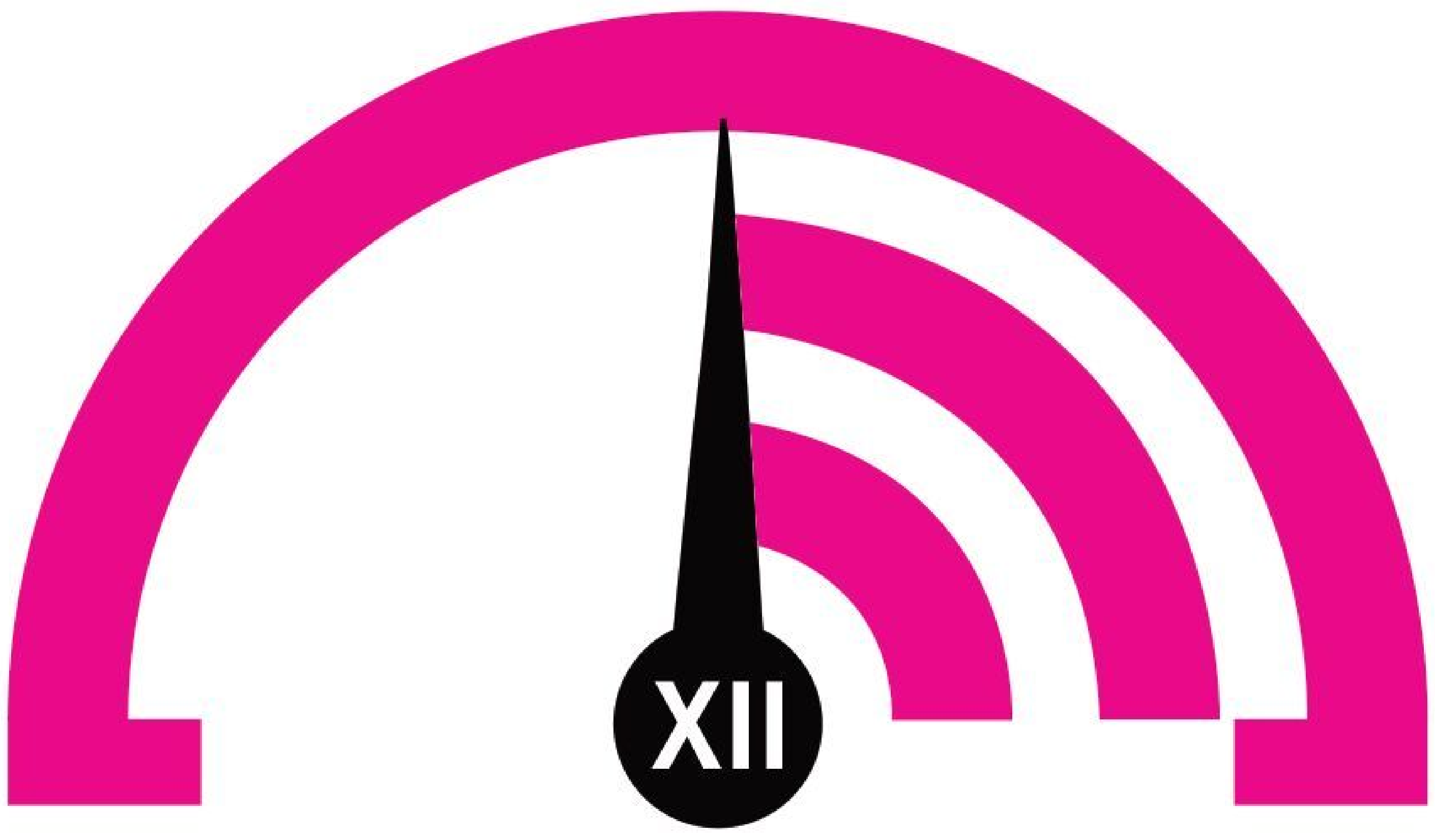
$$=\frac{\sqrt{3}}{2}+\frac{2\pi}{2}-\frac{\sqrt{3}}{2}-\frac{2\pi}{3}=\frac{\pi}{3}\ \text{sq. units}$$

◆ ◆

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1. (c)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (a)
6. (c)	**7.** (b,c)	**8.** (a,c)	**9.** (b,c)	**10.** (b,d)
11. (a,b,c,d)		**12.** (a,d)	**13.** (b)	**14.** (a)
15. (b)	**16.** (b)	**17.** (3)	**18.** (1)	**19.** (3)
20. (8)				

MONTHLY TEST
DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 9 : Vector Algebra/Three Dimensional Geometry

Time Taken : 60 Min.

Only One Option Correct Type

1. $ABCD$ a parallelelogram, and A_1 and B_1 are the midpoints of sides BC and CD, respectively. If $\overrightarrow{AB_1} + \overrightarrow{AA_1} = \lambda\overrightarrow{AC}$, the λ is equal to

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{3}{2}$ (d) 2

2. If λ is the length of any edge of a regular tetrahedron, then the distance of any vertex from the opposite face is

(a) $\frac{2}{3}\lambda^2$ (b) $\sqrt{\frac{2}{3}}\lambda$

(c) $\frac{\sqrt{2}}{3}\lambda$ (d) none of these

3. Vectors $3\vec{a}-5\vec{b}$ and $2\vec{a}+\vec{b}$ are mutually perpendicular. If $\vec{a}+4\vec{b}$ and $\vec{b}-\vec{a}$ are also mutually perpendicular, then the cosine of the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{19}{5\sqrt{43}}$ (b) $\frac{19}{3\sqrt{43}}$

(c) $\frac{19}{2\sqrt{45}}$ (d) $\frac{19}{6\sqrt{43}}$

4. The equation $ax + a^2y + z = 0$, $bx + b^2y + z = 0$ and $cx + c^2y + z = 0$ have only solution (0, 0, 0). The coefficient a, b, c are in G.P., then the common ratio of G.P. cannot be equal to

(a) 0, ± 1 (b) 2, ± 3

(c) 4, ± 5 (d) none of these

5. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1, 1 and 2 respectively. If $\vec{a}\times(\vec{a}\times\vec{c})+\vec{b}=0$, then the acute angle between $\vec{a}$ and $\vec{c}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{5\pi}{12}$

6. The line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the curve $xy = c^2$, $z = 0$, if c is equal to

SAMURAI SUDOKU

SOLUTION - FEBRUARY 2022

4	9	1	8	3	5	7	6	2				4	6	8	7	3	1	9	2	5
7	2	8	1	6	4	5	9	3				3	5	2	9	6	4	1	7	8
5	3	6	9	7	2	1	4	8				9	1	7	2	5	8	6	4	3
3	7	9	6	1	8	4	2	5				7	8	5	1	2	9	3	6	4
1	8	5	2	4	3	6	7	9				1	9	4	3	7	6	5	8	2
2	6	4	5	9	7	3	8	1				2	3	6	8	4	5	7	1	9
9	1	5	7	5	6	8	3	4	9	5	7	6	2	1	4	9	3	8	5	7
8	4	7	3	2	1	9	5	6	2	4	1	8	7	3	5	1	2	4	9	6
6	5	3	4	8	9	2	1	7	8	3	6	5	4	9	6	8	7	2	3	1
						1	9	2	6	8	4	3	5	7						
						5	7	8	1	2	3	9	6	4						
						4	6	3	5	7	9	1	8	2						
2	9	3	4	1	6	7	8	5	3	9	2	4	1	6	2	9	3	8	7	5
6	4	8	7	5	9	3	2	1	4	6	5	7	9	8	4	5	1	6	2	3
5	7	1	2	8	3	6	4	9	7	1	8	2	3	5	6	7	8	9	1	4
9	8	2	6	7	1	5	3	4				9	5	2	8	1	6	4	3	7
7	3	5	8	9	4	1	6	2				3	6	7	9	4	2	1	5	8
4	1	6	3	2	5	9	7	8				1	8	4	7	3	5	2	6	9
1	2	7	9	6	8	4	5	3				5	2	9	1	8	7	3	4	6
8	5	4	1	3	7	2	9	6				8	7	1	3	6	4	5	9	2
3	6	9	5	4	2	8	1	7				6	4	3	5	2	9	7	8	1

(a) ± 1 (b) $\pm \frac{1}{3}$

(c) $\pm \sqrt{5}$ (d) none of these

One or More than One Option(s) Correct Type

7. A vector $\vec{a}$ has the components $2p$ and 1 w.r.t. a rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counterclockwise sense. If with respect to a new system, $\vec{a}$ has components $(p + 1)$ and 1, the p is equal to______.

(a) -1 (b) $-1/3$

(c) 1 (d) 2

8. The coordinates of a point on the line $\frac{x-1}{2}=\frac{y+1}{-3}=z$ at a distance $4\sqrt{14}$ from the point $(1, -1, 0)$ are

(a) $(9, -13, 4)$

(b) $\left(8\sqrt{14}+1, -12\sqrt{14}-1, 4\sqrt{14}\right)$

(c) $(-7, 11, -4)$

(d) $\left(-8\sqrt{14}+1, -12\sqrt{14}-1, -4\sqrt{14}\right)$

9. Vectors $\vec{A}$ and $\vec{B}$ satisfying the vector equation $\vec{A}+\vec{B}=\vec{a}$, $\vec{A}\times\vec{B}=\vec{b}$ and $\vec{A}\cdot\vec{a}=1$, where $\vec{a}$ and $\vec{b}$ are given vectors, are

(a) $\vec{A}=\frac{\left(\vec{a}\times\vec{b}\right)-\vec{a}}{a^2}$

(b) $\vec{B}=\frac{\left(\vec{b}\times\vec{a}\right)+\vec{a}\left(a^2-1\right)}{a^2}$

(c) $\vec{A}=\frac{\left(\vec{a}\times\vec{b}\right)+\vec{a}}{a^2}$

(d) $\vec{B}=\frac{\left(\vec{b}\times\vec{a}\right)-\vec{a}\left(a^2-1\right)}{a^2}$

10. Let A be vector parallel to the line of intersection of planes P_1 and P_2 through origin. P_1 is parallel to the vector $2\hat{j}+3\hat{k}$ and $4\hat{j}-3\hat{k}$ and P_2 is parallel to $\hat{j}-\hat{k}$ and $3\hat{i}+3\hat{j}$, then the angle between A and $2\hat{i}+\hat{j}-2\hat{k}$ can be

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) $\frac{3\pi}{4}$

11. Let the unit vector $\vec{a}$ and $\vec{b}$ are perpendicular and the unit vector $\vec{c}$ inclined at an angle α to $\vec{a}$ and $\vec{b}$. If $\vec{c}=l\vec{a}+m\vec{b}+n(\vec{a}\times\vec{b})$, then

(a) $l = m$ (b) $n^2 = 1 - 2l^2$

(c) $n^2 = -\cos 2\alpha$ (d) $m^2=\frac{1+\cos 2\alpha}{2}$

12. If the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{\lambda}$ and $\frac{x-1}{\lambda}=\frac{y-4}{2}=\frac{z-5}{1}$ intersect, then

(a) $\lambda = -1$ (b) $\lambda = 2$ (c) $\lambda = -3$ (d) $\lambda = 0$

Comprehension Type

Paragraph for Q. No. 13 to 15

From any point $P(a, b, c)$ are drawn perpendiculars PM and PN to ZX and XY planes respectively. Let α, β, γ be

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique solution.

			6												5					
		5		1										4		6				
		8			7	3	1							8			2	9	7	
		9		4				3						3		8				2
	5		8		9		4						7		1		5		3	
4				3		9						2				7		6		
	3	1	4			2			8				5	7	2			8		
				7		5		8		2						3		5		
					6			1			5	8	2				6			
								5		6				4						
							9		4		7		6							
						6				5		9								
			8				5	4	2			1			6					
		9		2						7		4		2		1				
		6			3	9	1				8			6			9	2	8	
		4		7				9						1		5				2
	7		2		8		4						4		7		1		9	
5				4		6						8				4		3		
	9	7	6			4							1	9	2			6		
				8		2										7		9		
					5												4			

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the angles which *OP* makes with coordinate planes and θ be the angle which *OP* makes with the plane *OMN*.

13. Equation of the plane *OMN* be

(a) $\frac{x}{a}-\frac{y}{b}+\frac{z}{c}=0$

(b) $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}=0$

(c) $-\frac{x}{a}-\frac{y}{b}+\frac{z}{c}=0$

(d) none of these

14. $\sin\theta$ must be equal to

(a) $\frac{abc}{\sqrt{a^2+b^2+c^2}\sqrt{a^2b^2+b^2c^2+c^2a^2}}$

(b) $\frac{ab+bc+ca}{a^2+b^2+c^2}$

(c) $\frac{a+b+c}{\sqrt{a^2+b^2+c^2}}$

(d) none of these

15. $\operatorname{cosec}^2\theta$ must be equal to

(a) $\cot^2\alpha + \cot^2\beta + \cot^2\gamma$

(b) $\operatorname{cosec}^2\alpha + \operatorname{cosec}^2\beta + \operatorname{cosec}^2\gamma$

(c) $\operatorname{cosec}\alpha + \operatorname{cosec}\beta + \operatorname{cosec}\gamma$

(d) none of the above

Matrix Match Type

16. Match the following :

	Column-I		Column-II
P.	If $\vec{a}+\vec{b}=\hat{j}$ and $2\vec{a}-\vec{b}=3\hat{i}+\frac{\hat{j}}{2}$, then cosine of the angle between $\vec{a}$ and $\vec{b}$ is	1.	1
Q.	If angle between each pair of vectors is $\frac{\pi}{3}$ and $\lvert\vec{a}+\vec{b}+\vec{c}\rvert=\sqrt{6}$, then $\lvert\vec{a}\rvert=$	2.	$5\sqrt{3}$
R.	Area of the parallelogram whose diagonals represents the vectors $3\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}-3\hat{j}+4\hat{k}$ is	3.	7
S.	If- $\vec{a}$ is perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ is perpendicular to $\vec{c}+\vec{a}$, $\vec{c}$ is perpendicular to $\vec{a}+\vec{b}$, $\lvert\vec{a}\rvert=2$, $\lvert\vec{b}\rvert=3$ and , then $\lvert\vec{a}+\vec{b}+\vec{c}\rvert=$	4.	$-\frac{3}{5}$

	P	Q	R	S
(a)	1	2	3	4
(b)	4	1	2	3
(c)	2	3	1	1
(d)	2	1	3	2

Numerical Answer Type

17. Let $\vec{u}=\hat{i}+\hat{j}$, $\vec{v}=\hat{i}-\hat{j}$ and $\vec{w}=\hat{i}+2\hat{j}+3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}\cdot\hat{n}$ and $\vec{v}\cdot\hat{n}=0$, then $\lvert\vec{w}\cdot\hat{n}\rvert$ is equal to______.

18. If the planes $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ pass through a straight line, then find the value of $a^2 + b^2 + c^2 + 2abc$_______.

19. Let $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=2\vec{a}+10\vec{b}$, $\overrightarrow{OC}=\vec{b}$ where *O, A, C* are non-collinear points. Let l denote the area of the quadrilateral *OABC*. Let m denotes the area of parallelogram with $\overrightarrow{OA}$ and $\overrightarrow{OC}$ as adjacent sides. If $l = 2\lambda m$, then the value of λ is ____.

20. Find the number of points, whose perpendicular distances from *yz*, *zx* and *xy*-planes are in A.P. and whose distances from *x*, *y* and *z*-axes are $\sqrt{13}$, $\sqrt{10}$ and $\sqrt{5}$ respectively.

◆◆

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